Section 16
Fundamentals of Microgravity Vibration Isolation

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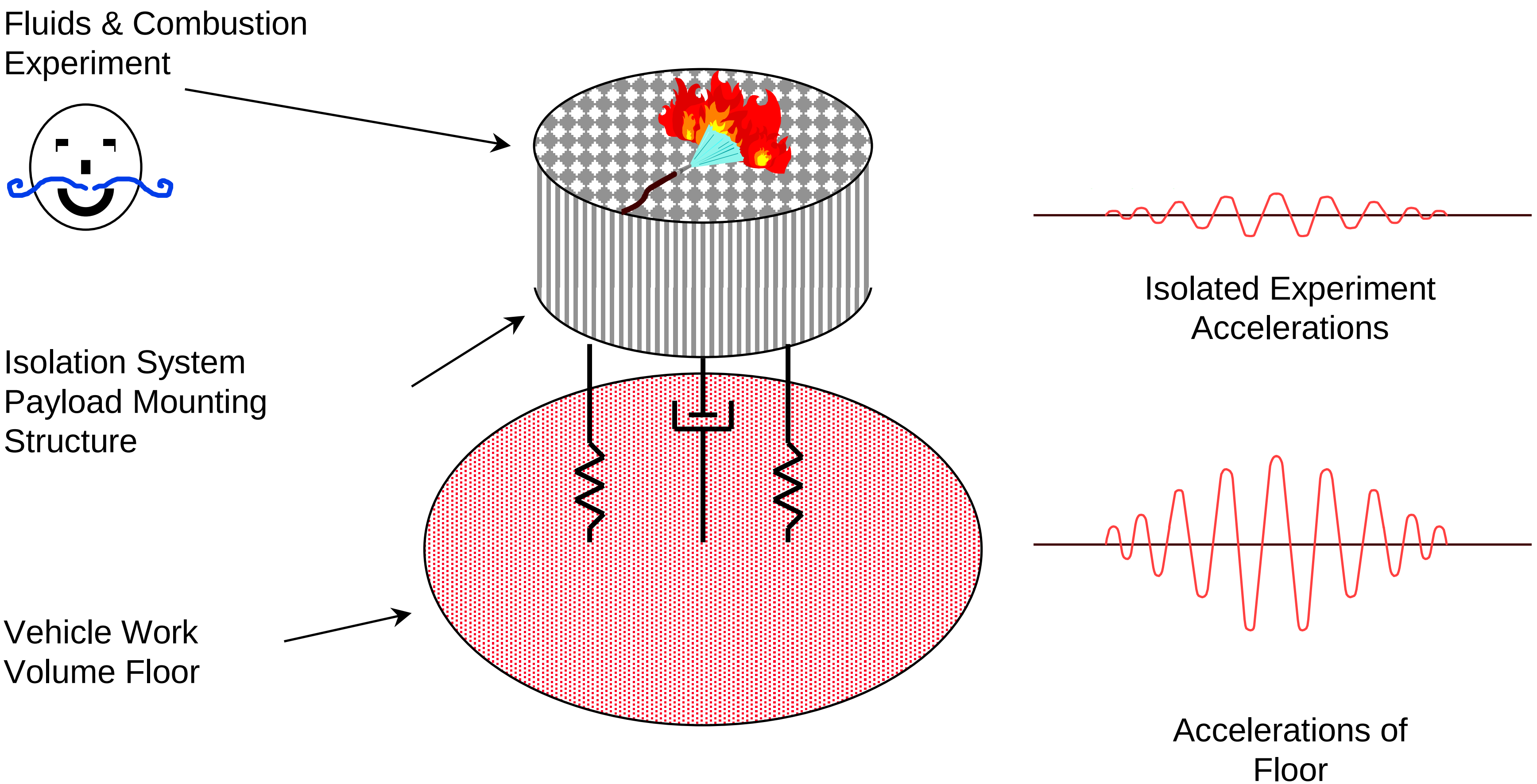
Outline:

- **Motivation**
- **Dynamics of Systems**
- **Active Control Concepts**
- **Active Control Examples**
- **Modern Control Approaches**

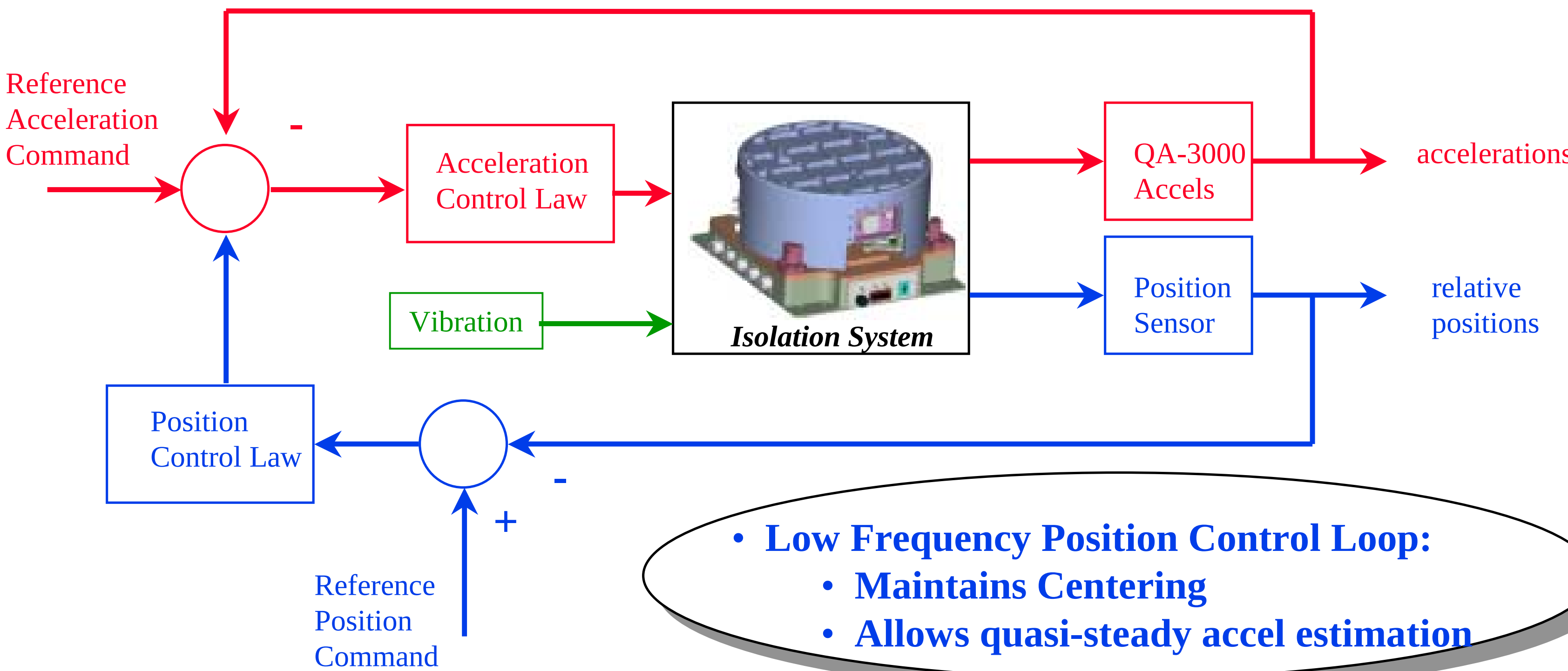
Introduction

- The ambient spacecraft acceleration levels are often higher than allowable from a science perspective.
- To reduce the acceleration levels to an acceptably quiescent level requires vibration isolation.
- Either passive or active isolation can be used depending on the needs or requirements of a specific application.

What is Vibration Isolation?

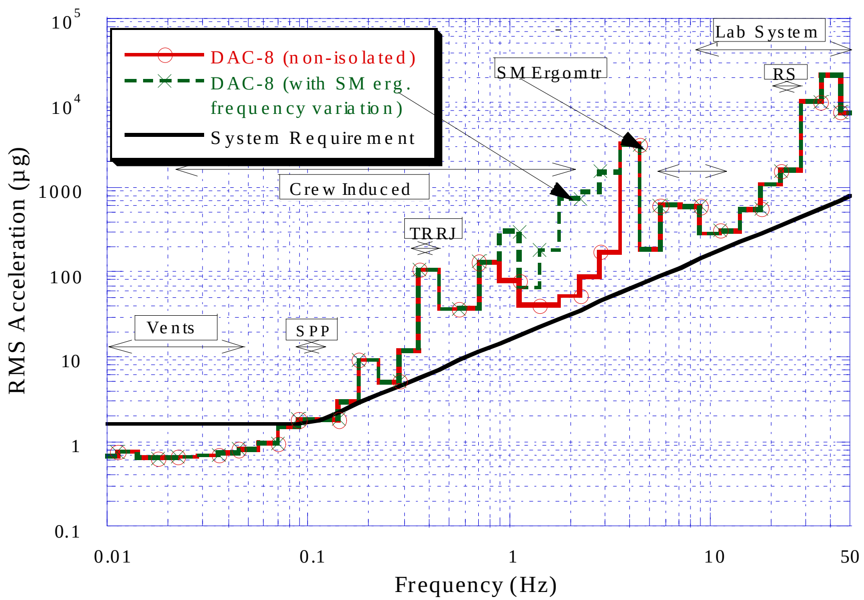


- **High Frequency Acceleration Control Loop:**
  - **Cancels Inertial Motion of the Platform**
  - **Allows “Good Vibrations”**



- **Low Frequency Position Control Loop:**
  - **Maintains Centering**
  - **Allows quasi-steady accel estimation**

Why is Vibration Isolation Needed?



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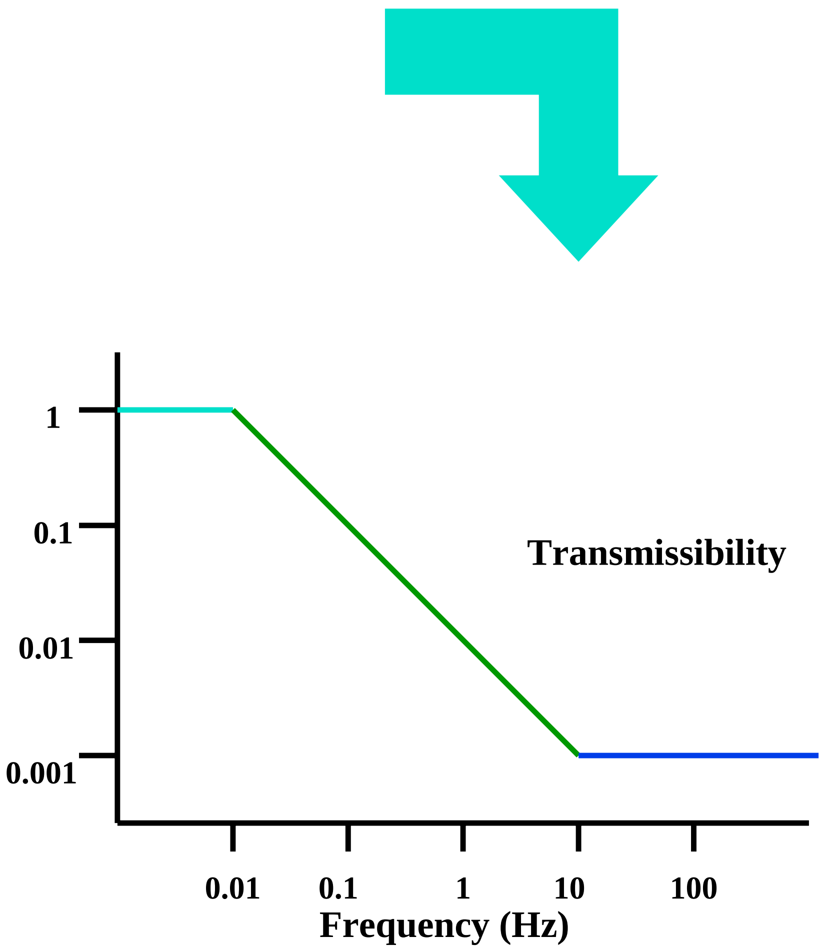
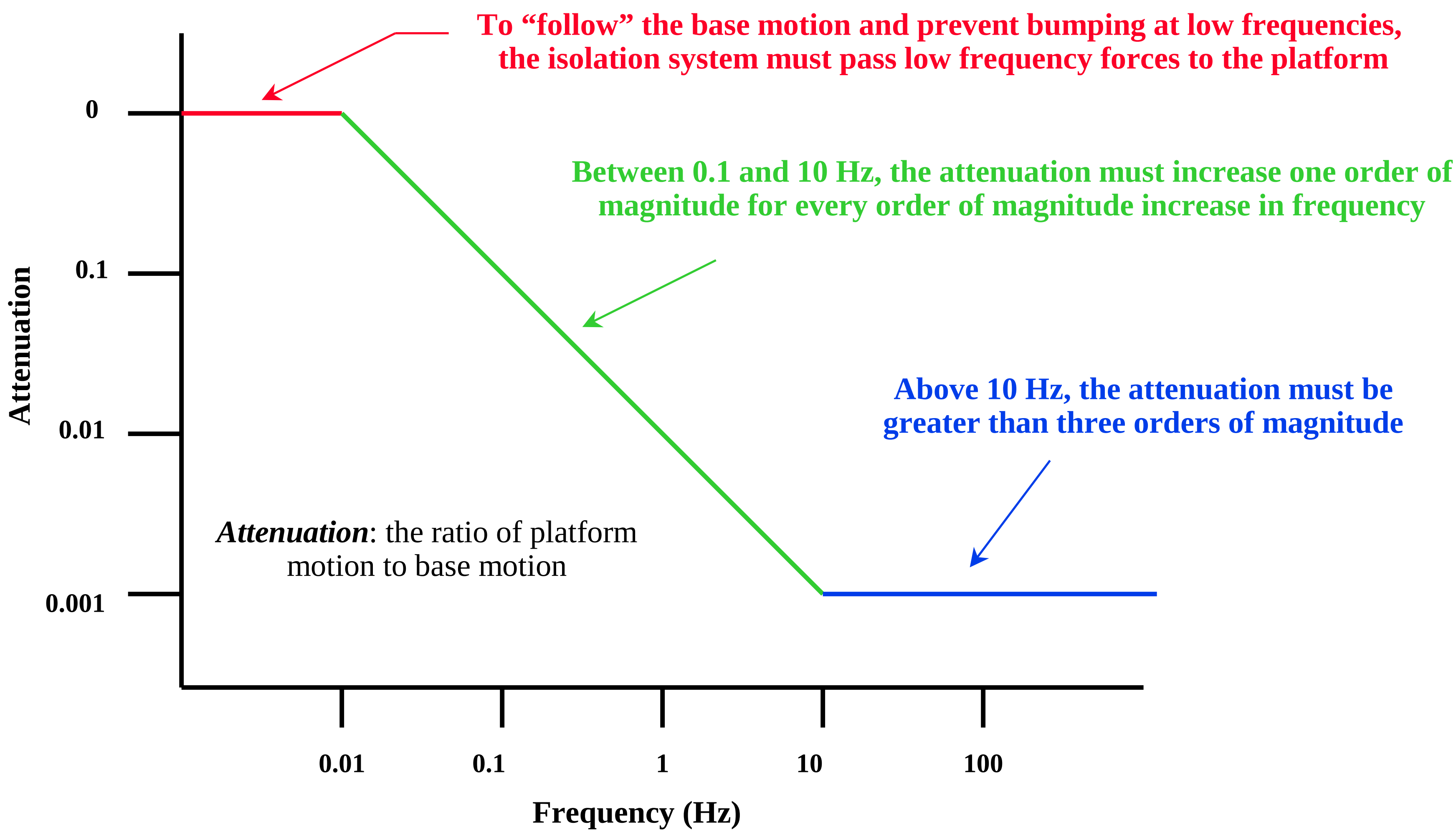
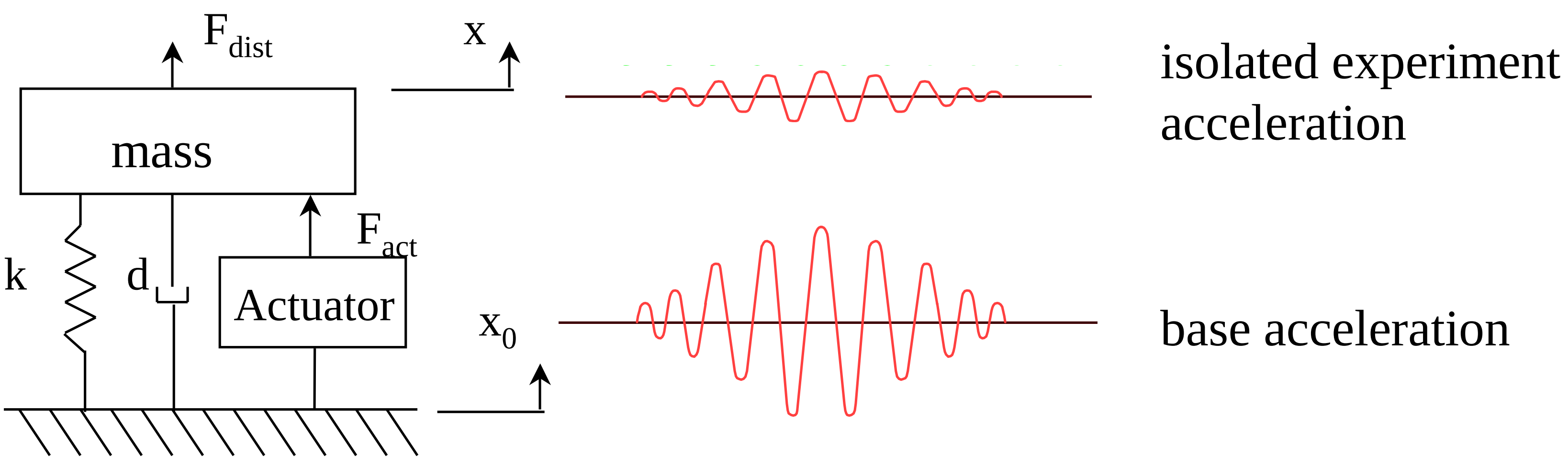


Attenuation Requirement



Single Degree Of Freedom (DOF) Example: Spring-Mass-Damper

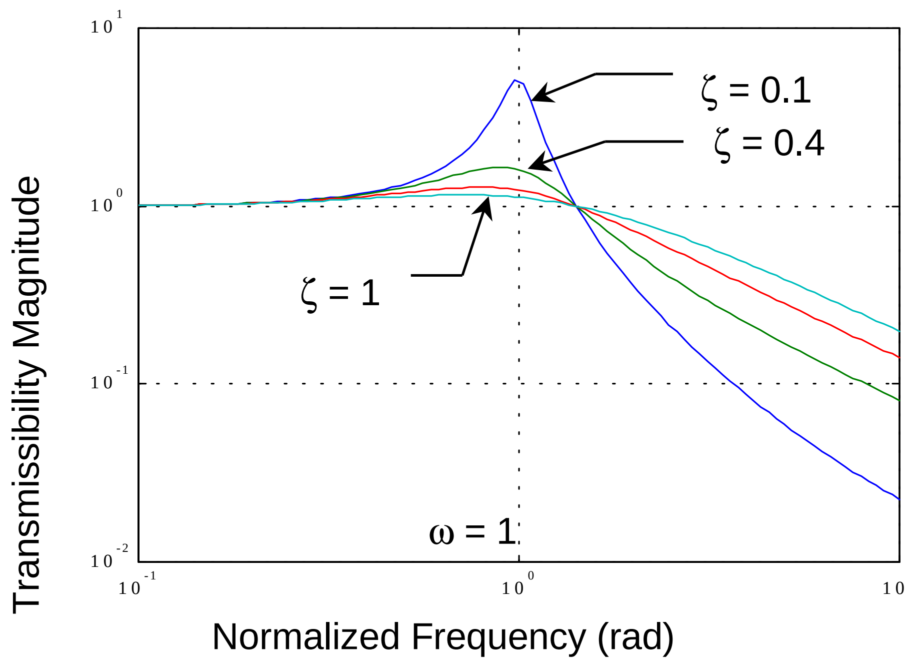


Equation of motion: $m\ddot{x} + d(\dot{x} - \dot{x}_0) + k(x - x_0) = F_{dist} + F_{act}$

The dynamic response of the mass to a base acceleration is a function of the system mass, stiffness, and damping.

System Dynamics: Transmissibility

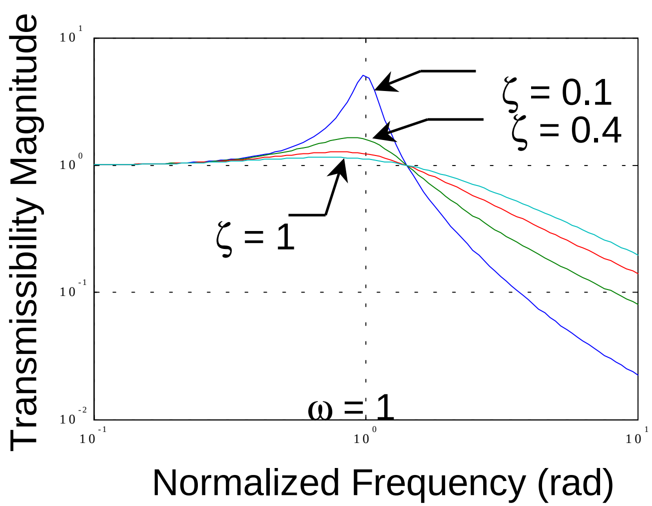
Transmissibility is the magnitude of the transfer function relating the acceleration (or position) of the mass to the base acceleration (or position). The transmissibility specifies the attenuation of base motion as a function of frequency.



Passive Vibration Isolation

- Select spring stiffness, mass, and damping for attenuation
- Reduce break frequency by minimizing spring stiffness

  Typically not desirable to increase isolated mass
- Select damping to trade between damped resonance and rate of attenuation



Transmissibility: $$\frac{x}{x_0} = \frac{2\zeta\omega s + \omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

Natural Frequency: $$\omega = \sqrt{\frac{k}{m}}$$

Damping Ratio: $$\zeta = \frac{d}{2\sqrt{km}}$$

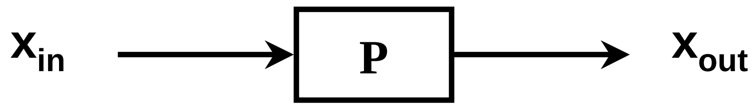
Active Vibration Isolation

- Reduce the inertial motion of payload by sensing motion and applying forces to counter measured motion
- Active control can effectively change the system mass, stiffness, and damping ***as a function of frequency***
- Whereas passive isolation only attenuates forces in passive elements, active control attenuates measured motion
  - Only active control can mitigate payload response to payload-induced vibrations
- Requires power, sensors, actuators, control electronics (analog and digital)

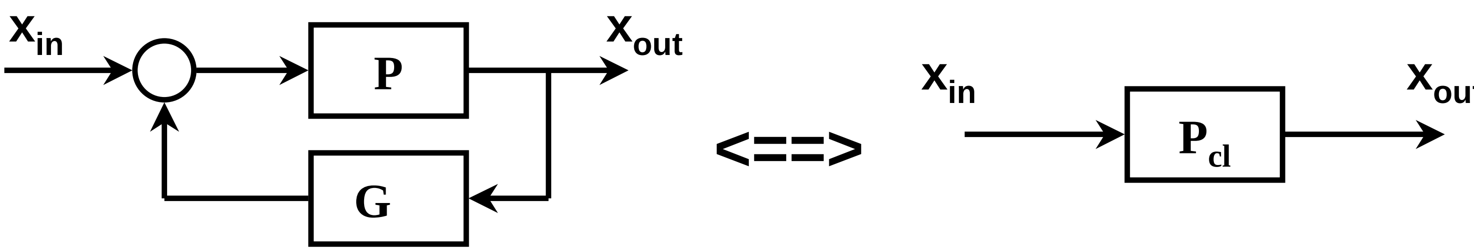
Active Control Illustration

Consider the transfer function from base position to mass displacement:

$$P = \frac{ds + k}{ms^2 + ds + k}$$



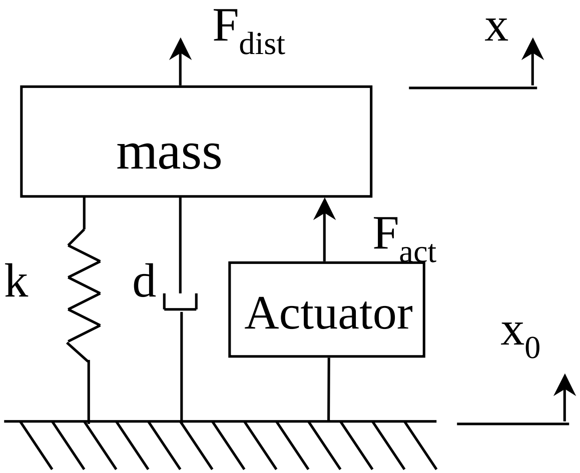
Now measure the displacement and "feed it back" with gains (K_a, K_v, K_p) and a control law given by $G = - K_a s^2 - K_v s - K_p$



The closed loop transfer function becomes:

$$P_{cl} = \frac{ds + k}{\underbrace{(m+K_a)s^2}_{\tilde{\mathbf{m}}} + \underbrace{(d+K_v)s}_{\tilde{\mathbf{d}}} + \underbrace{(k+K_p)}_{\tilde{\mathbf{k}}}}$$

Active Isolation Example



Recall the Spring-Mass-Damper Example

Equation of motion:

$$m\ddot{x} + d(\dot{x} - \dot{x}_0) + k(x - x_0) = F_{dist} + F_{act}$$

Consider the control law: $F_{act} = -K_a \ddot{x} - K_v(\dot{x} - \dot{x}_0) - K_p(x - x_0)$

The resulting closed loop transmissibility is:

$$\frac{x}{x_0} = \frac{2\varsigma_{cl}\omega_{cl}s + \omega_{cl}^2}{s^2 + 2\varsigma_{cl}\omega_{cl}s + \omega_{cl}^2}$$

and the closed loop natural frequency and damping become:

$$\omega_{cl} = \sqrt{\frac{k + K_p}{m + K_a}} \qquad \varsigma_{cl} = \frac{(d + K_v)}{2\sqrt{(k + K_p)(m + K_a)}}$$

	Passive Isolation	**Active Isolation**
Transmissibility:	$\frac{x}{x_0} = \frac{2\varsigma\omega s + \omega^2}{s^2 + 2\varsigma\omega s + \omega^2}$	$\frac{x}{x_0} = \frac{2\varsigma_{cl}\omega_{cl}s + \omega_{cl}^2}{s^2 + 2\varsigma_{cl}\omega_{cl}s + \omega_{cl}^2}$
Natural Frequency:	$\omega = \sqrt{\frac{k}{m}}$	$\omega_{cl} = \sqrt{\frac{k + K_p}{m + K_a}}$
Damping Ratio:	$\varsigma = \frac{d}{2\sqrt{km}}$	$\varsigma_{cl} = \frac{(d + K_v)}{2\sqrt{(k + K_p)(m + K_a)}}$

Active Control Concepts

However, it isn't as easy as it seems --

- Real systems aren't simple one degree of freedom lumped masses with discrete springs and dampers.

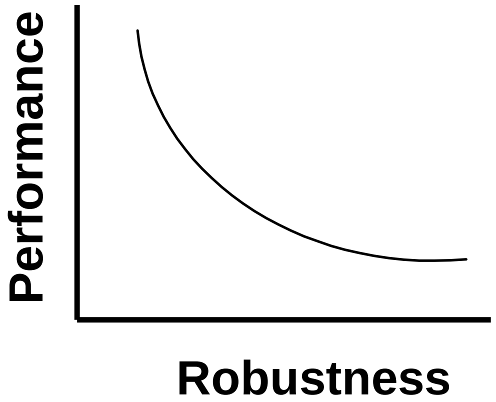
- Control system design is a function of system properties which typically aren't well known.

The two key control design issues are ***performance*** and ***robustness***.

- ***Performance***: how well is isolation achieved?
- ***Robustness***: how well are uncertainties tolerated by the control system?

Key Control Issues

Robustness and **Performance** of a closed loop system are ***always*** in opposition



- Robustness to uncertainties:
  - umbilical properties
  - structural flexibility
  - mass and inertia variations
  - sensor & actuator dynamics
- Performance:
  - base motion attenuation
  - payload disturbances
  - forced excitation

Control Challenges

- Robustness to uncertainties:
  - umbilical properties
  - structural flexibility
  - mass and inertia variations
  - sensor & actuator dynamics

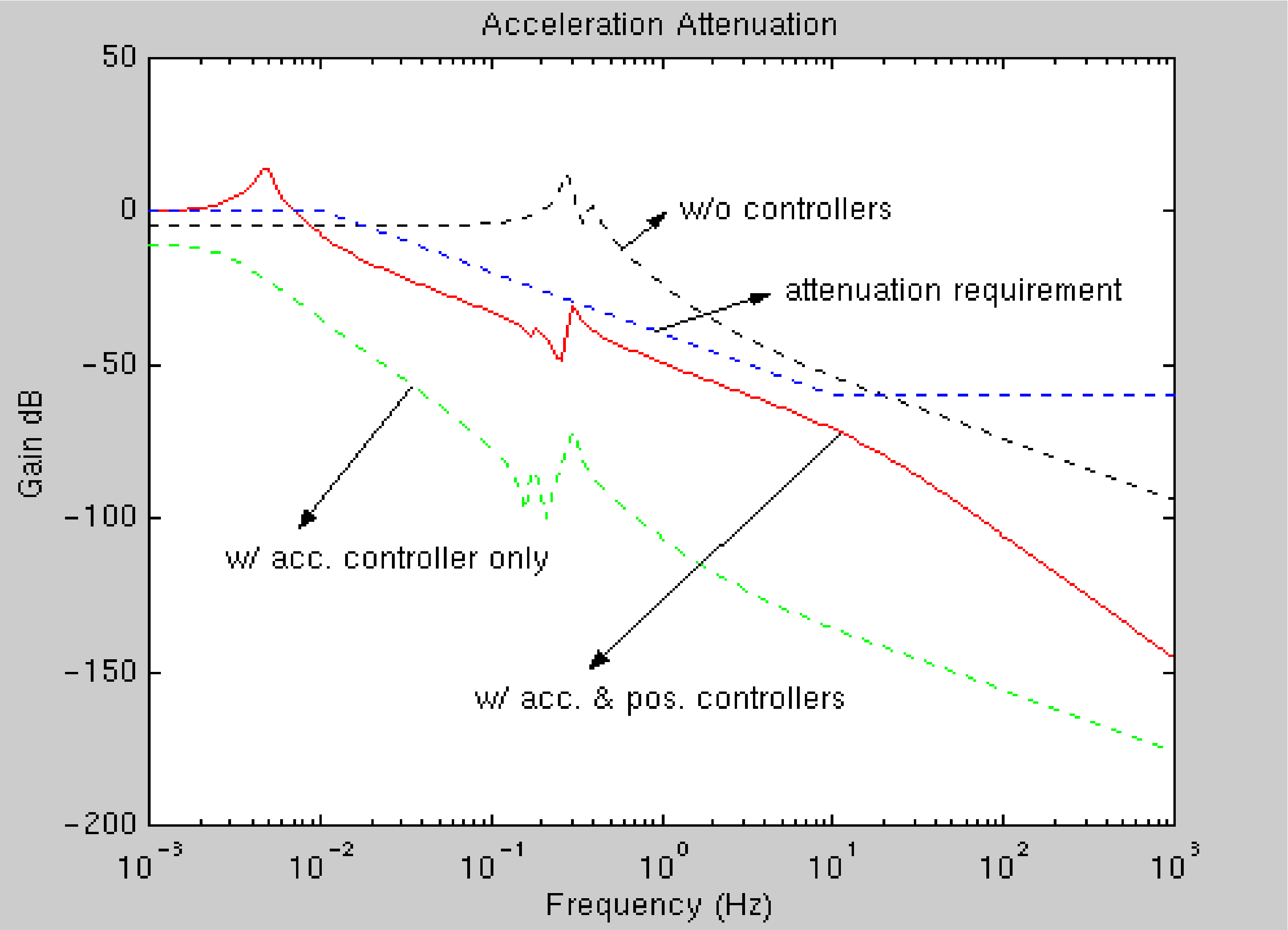
Low Gain &/or Low Bandwidth

- Performance:
  - base motion attenuation
  - payload disturbances
  - forced excitation

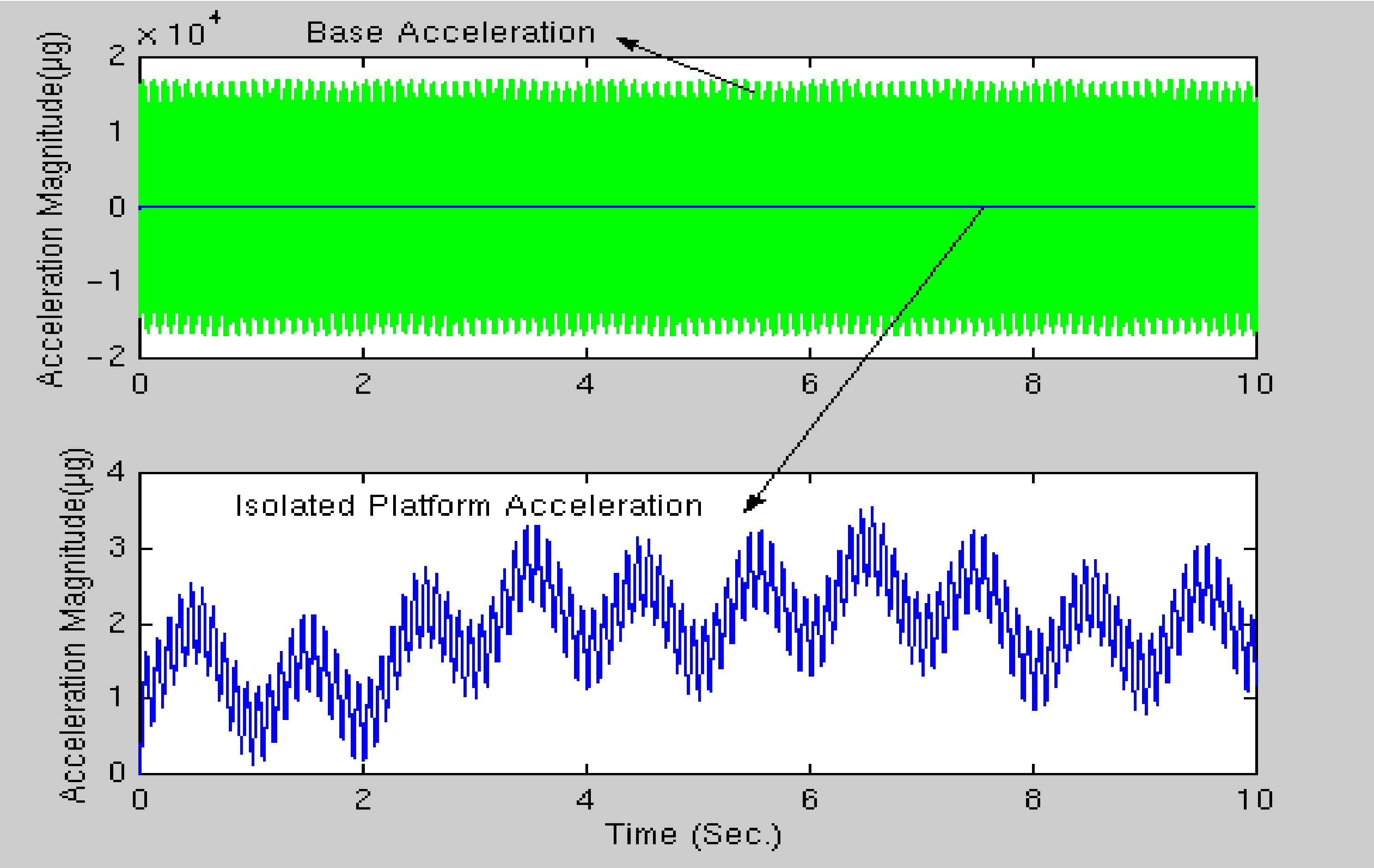
High Gain

High Bandwidth

g-LIMIT 6DOF, Baseline PID Controllers (X-axis)

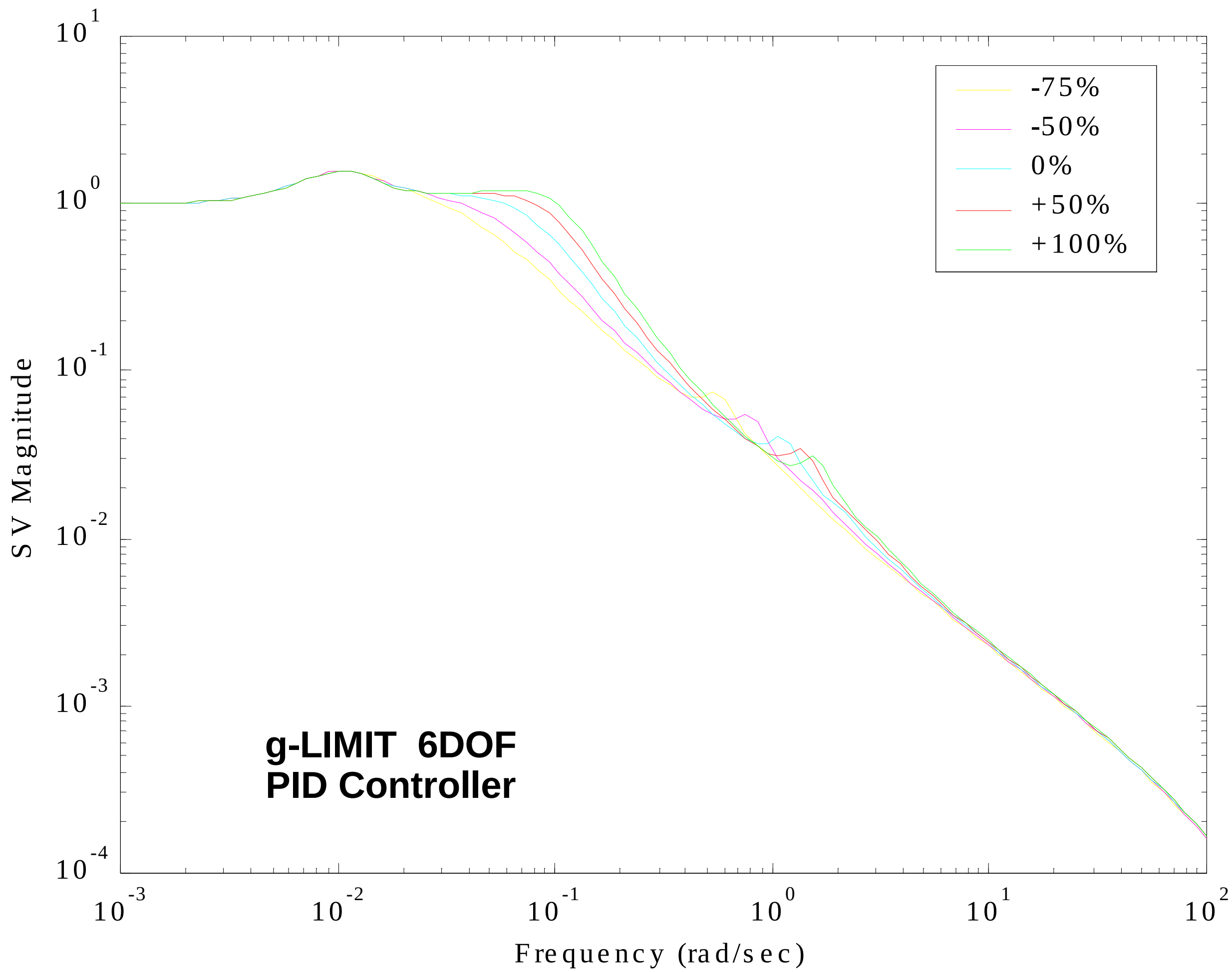


g-LIMIT 6DOF, Acceleration Time Response (X-axis)

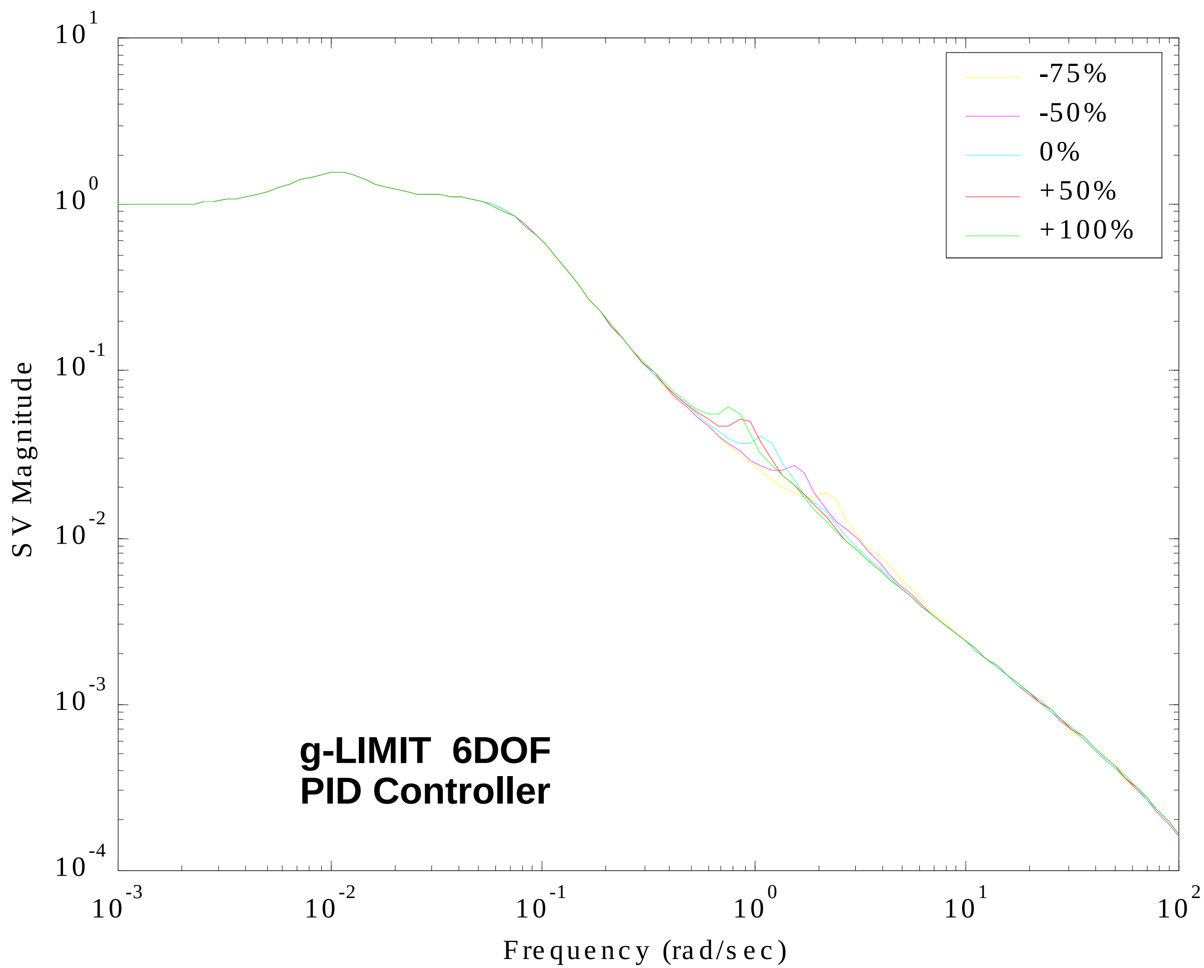


Base acceleration = 1.6 sin(0.01 hz*t)+16 sin(0.1 hz*t)+160 sin(1 hz*t)+1600 sin(10 hz*t)+16000 sin(100 hz*t)

Transmissibility Max Singular Value with Stiffness Variation



Transmissibility Max Singular Value with Mass/Inertia Variation



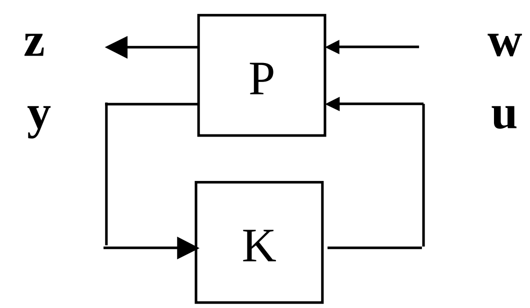
Modern Control Approaches
to
Microgravity Vibration Isolation

Robust multivariable microgravity vibration control systems maximize performance for a specified bounded set uncertainties

Design for Nominal Performance (NP): H_2 Methods

- Good nominal performance
- Performance metric well suited for μg vibration isolation
- Very poor robustness
- High order controllers

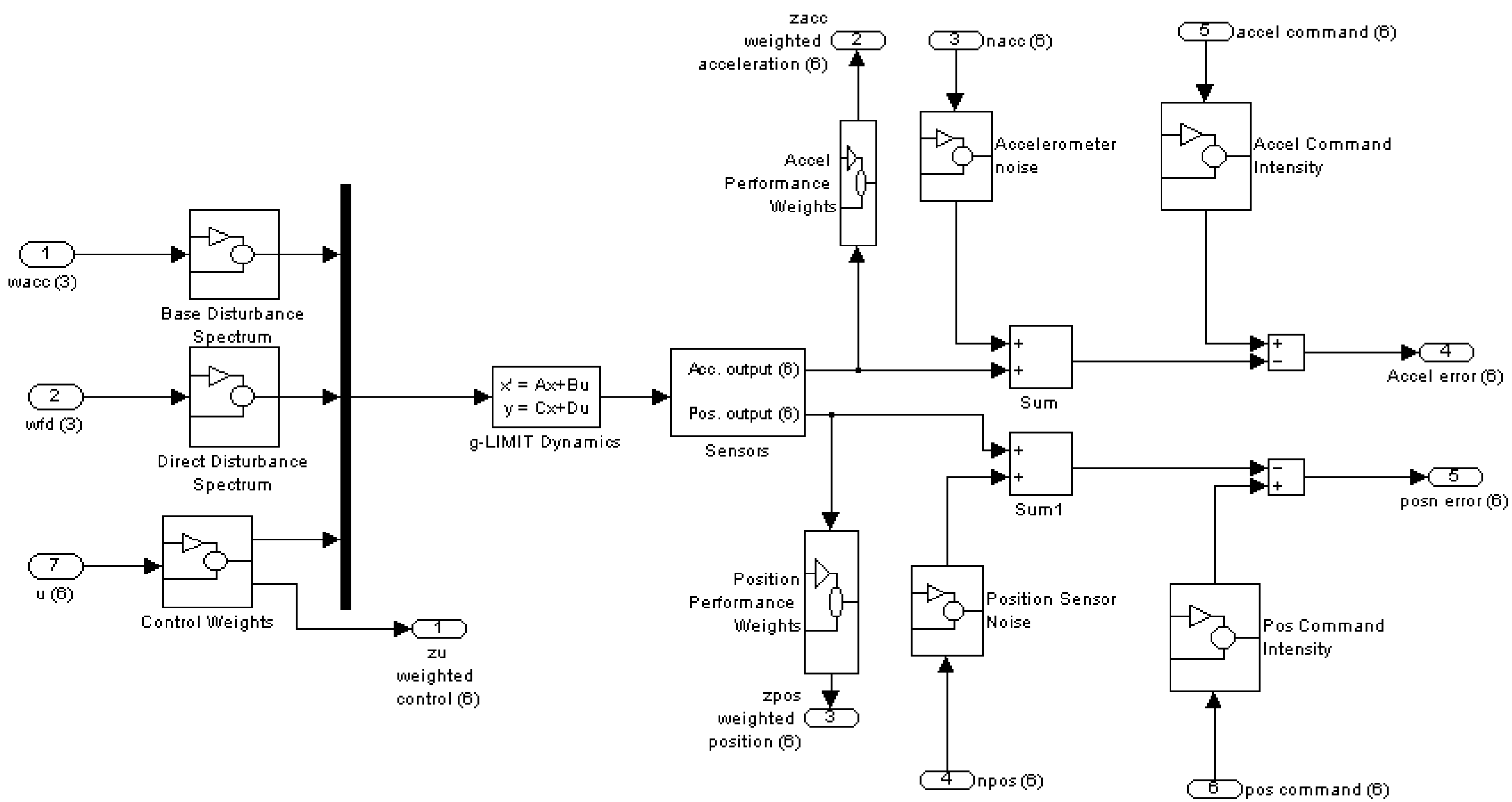
$$K_2 = \arg\left\{\min_{K} \left\| T_{zw} \right\|_2\right\}$$



where $$\left\| T_{zw} \right\|_2 = \lim_{t \to \infty} E\{z(t)^T z(t)\}$$

$$K_2 : \left\{ \begin{array}{c} \dot{x}_c = A_c x_c + B_c y \\ u = C_c x_c \end{array} \right\} \qquad x_c \in \Re^{nc}$$

Generalized Plant for H2 Design



g-LIMIT H_2 Control Design

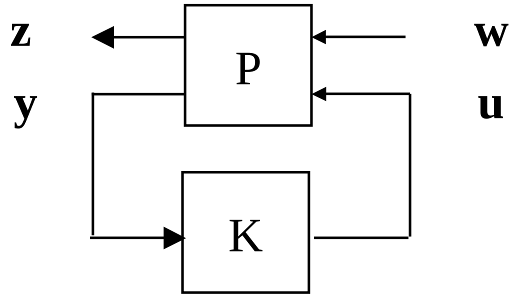
- **Objective: minimize H_2 norm of closed loop from disturbances, w, to performance variables, z**

w =
base acceleration
payload induced force
accelerometer noise
position sensor noise

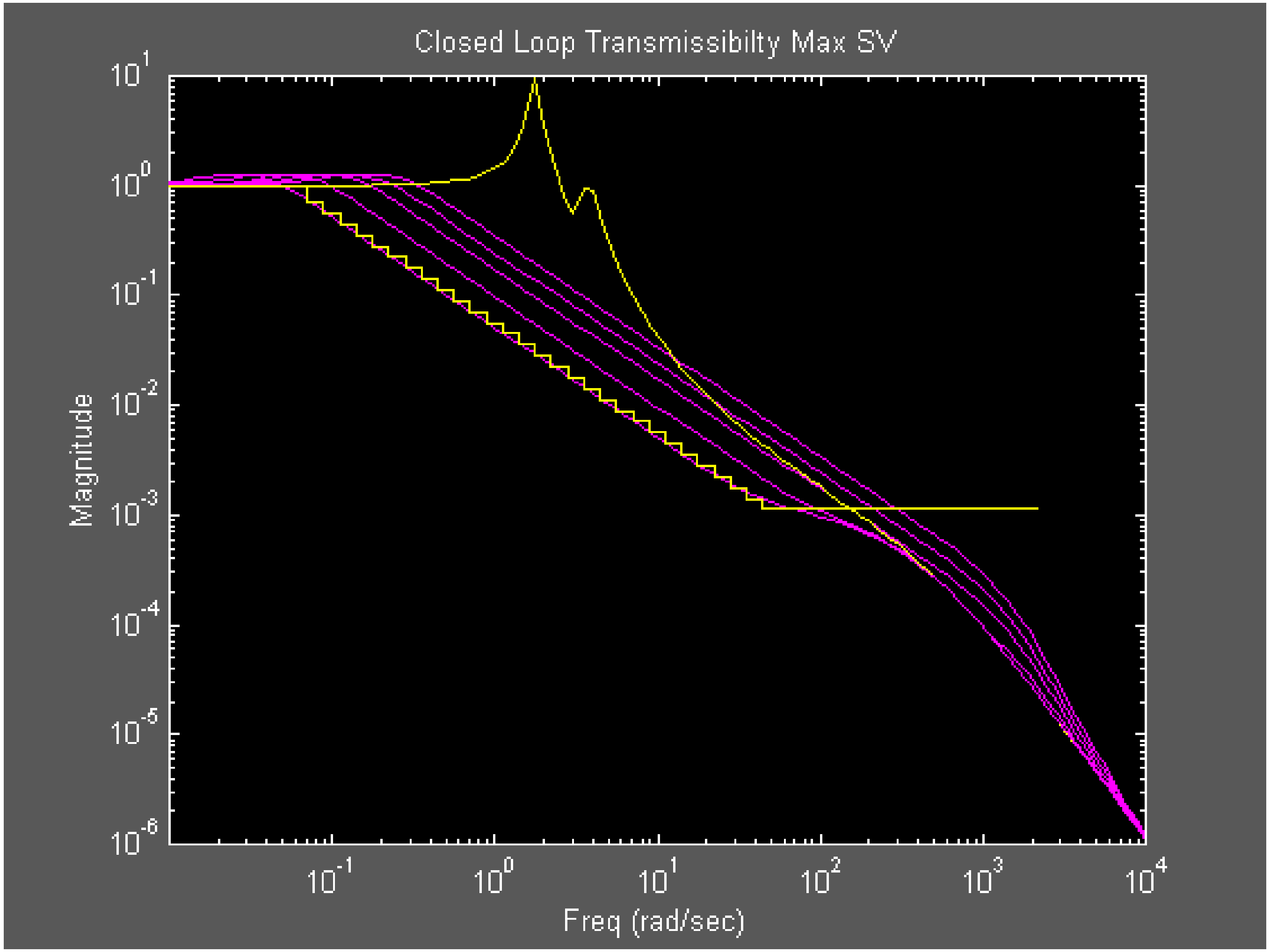
z =
weighted control
weighted acceleration
weighted relative position

y =
platform acceleration
relative position

u = control actuators



g-LIMIT 6 DOF H2 Design Performance



Design for Robust Stability (RS): H_∞ Methods:

- Sufficient condition for RS of all plants in the set parameterized by the bounded model errors

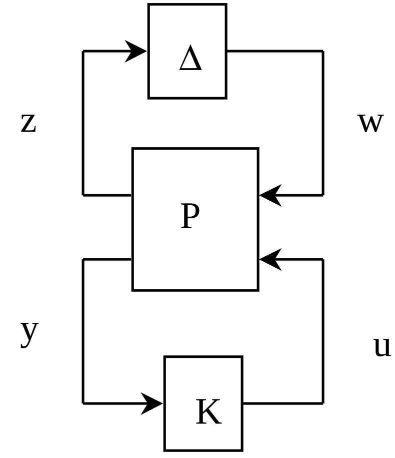
$\Delta \in \Delta_\delta, \Delta_\delta = \{\Delta : \|\Delta\|_\infty < \delta\}$ is $\|T_{zw}\|_\infty < \frac{1}{\delta}$

- Performance metric is the peak magnitude of transfer function – not well suited for μg vibration isolation
- High order controllers

$$K_\infty = \arg\left\{\min_K \|T_{zw}\|_\infty\right\}$$

where

$$\|T_{zw}\|_\infty = \sup_\omega \left\{\bar{\sigma}(T_{zw}(j\omega))\right\}$$



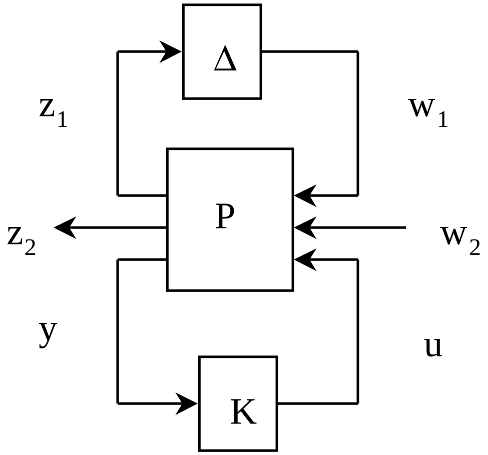
Design for Nominal Performance and Robust Stability

Mixed H_2 /H_∞ Methods:

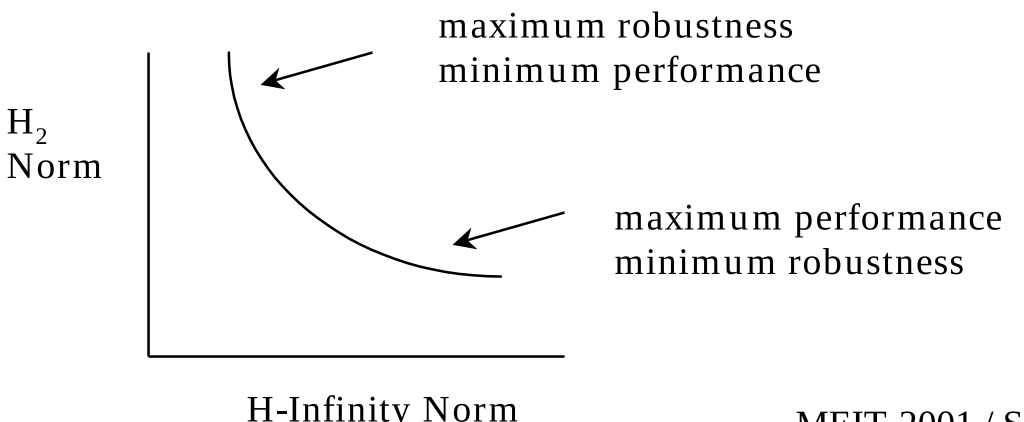
- Optimizes H_2 nominal performance
- Guarantees H_∞ robust stability
- Optimized controller of FIXED DIMENSION
- Extremely computationally intensive
- Objective:
  - NP - $\min \left\| T_{z2w2} \right\|_2$

  Subject to

  - RS - $\left\| T_{z1w1} \right\|_\infty < \delta$



- The utility of mixed norm design is exploited by separating performance and robustness using the most appropriate norms
- A set of controllers is designed that explicitly trades between RS and NP
- Determine maximum achievable performance subject to robust stability constraints



Where Do We Go From Here?

First generation isolation systems are currently in flight demonstration phase

Once operational, will require significant sustaining engineering

- payload scheduled control design
- routine ongoing performance/stability analysis
- loss of science time

Second generation systems should provide better isolation performance in a more cost effective manner

- maximize isolation performance
- minimize payload impacts
- autonomous operation & optimization

Neural Network Based Adaptive Control Systems

Accommodate payload uncertainties/variations

- mass/inertia
- structural modes
- center of gravity

Biologically inspired technology

- autonomous adaptation
- reduces sustaining engineering
- maximize isolation performance

Significant technology transfer potential

Demonstrated in various aerospace vehicle applications

Further Reading:

1. Grodsinsky C. and Whorton, M., "Survey of Active Vibration Isolation Systems for Microgravity Applications," *Journal of Spacecraft and Rockets*, Vol. 37, No. 5, Sept. – Oct. 2000.
2. Knospe, C. R., Hampton, R. D., and Allaire, P. E., "Control Issues of Microgravity Vibration Isolation," *Acta Astronautica*, Vol. 25, No. 11, 1991, pp. 687-697.
3. Kuo, Benjamin C., Automatic Control Systems, Prentice-Hall, 1987
4. Thomson, William T., Theory of Vibration With Applications, Prentice-Hall, 1988.