Section 13

Analysis Techniques for Vibratory Data

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Acronyms & Abbreviations

µg	micro-g, 10^{-6}g (sometimes ug – "you gee")
AM	Ante Meridiem
ARIS	Active Rack Isolation System
ATL	Attitude Time Line
AVG	Average (mean) value
CD	Compact Disc
CDT	Central Daylight Time
CEVIS	Cycle Ergometer with Vibration Isolation System
CMG	Control Moment Gyro
DC	Direct Current (mean value)
DFT	Discrete Fourier Transform
dpi	dots per inch
EDT	Eastern Daylight Time
ER	EXPRESS Rack
EVA	Extravehicular Activity
EXPRESS	Expedite the Processing of Experiments to the Space Station
FE	Flight Engineer
FFT	Fast Fourier Transform
FGB	Functionalui Germatischeskii Block
ft	feet
g	acceleration due to free-fall (9.81 m/s^2)
GASMAP	Gas Analysis System for Metabolic Analysis of Physiology
GMT	Greenwich Mean Time
GRC	Glenn Research Center
HRF	Human Research Facility
Hz	Hertz
ISS	International Space Station
ITA	Instrumentation Technology Associates
JPG, JPEG	Joint Photographic Experts Group
JSC	Johnson Space Center
LAB	Laboratory
LAB1O1	US LAB Overhead 1
LAB1O2	US LAB Overhead 2
LAB1P3	US LAB Port 3
LAB1S1	US LAB Starboard 1
LAB1S2	US LAB Starboard 2
LAB1S3	US LAB Starboard 3
LMS	Life and Microgravity Spacelab Mission
LOS/AOS	Loss Of Signal/Acquisition Of Signal
LSLE	Life Sciences Laboratory Equipment
MAMS	Microgravity Acceleration Measurement System
MAX	Maximum value
MEC	Medical Equipment Computer
MEIT	Microgravity Environment Interpretation Tutorial
MEPS	Microencapsulation Electrostatic Processing System
MET	Mission Elapsed Time
mg	milli-g, 10^{-3}g
MIN	Minimum value
MSG	Microgravity Science Glovebox
OARE	Orbital Acceleration Research Experiment
OSS	OARE Sensor Subsystem
OTO	One-Third Octave
PAD	PIMS Acceleration Data
PAO	Public Affairs Office
PC	Personal Computer
PCMCIA	Personal Computer Memory Card International Association
PCSA	Principal Component Spectral Analysis
PFE	Periodic Fitness Evaluation
PFMI	Pore Formation and Mobility Investigation
PIMS	Principal Investigator Microgravity Services
PSD	Power Spectral Density
PuFF	Pulmonary Function in Flight
RF, R/F	Refrigerator Freezer
RMS	Root-Mean-Square
RPM	Revolutions Per Minute
RTS	Remote Triaxial Sensor
SAMS	Space Acceleration Measurement System
SE	Sensor Enclosure
SKV	Russian Air Conditioner
SM	Service Module
SSA	Space Station Analysis
STS	Space Transportation System
SUBSA	Solidification Using a Baffle in Sealed Ampoules
TEA	Torque Equilibrium Attitude
TeSS	Temporary Sleep Station
TVIS	Treadmill Vibration Isolation System
UF	Utilization Flight
US	United States
VELO	Velosiped
XPOP	X Principal Axis Perpendicular to the Orbit Plane

Outline

- Overview
- Time Domain Analysis
  - Interval statistics
- Frequency Domain Analysis
  - Fourier Transform
    - Discrete Fourier Transform (DFT)
    - Fast Fourier Transform (FFT)
  - Power Spectral Density (PSD)
  - Spectral Averaging
  - Parseval's Theorem
  - Spectrogram
  - Principal Component Spectral Analysis (PCSA)
- Summary

Overview

Objectives:

- characterize significant traits of the measured data …………….. (qualify/quantify)
- compare measured data to history, requirements, or predictions
- summarize measured data

what was happening, when, how often, where?

how much? impact at other locations?

Motivations:

- assist investigators and maintain knowledge base
- provide feedback to those interested in a data set's relativity
- manage large data sets

how a specific data set of interest measures up to previous data, other locations, specifications, simulations, or models?

Approaches:

- time domain analysis
- frequency domain analysis

Time Domain Analysis

Objectives:

- isolate acceleration events with respect to time
- correlate acceleration data with other information (logs, timelines, plans)
- limit checking against science or vehicle requirement thresholds

Approaches:

- acceleration vs. time → **use interval statistics:**
  average (AVG)
  root-mean-square (RMS)
  minimum/maximum (MIN/MAX)

Time Domain Analysis
Acceleration vs. Time

Advantages:

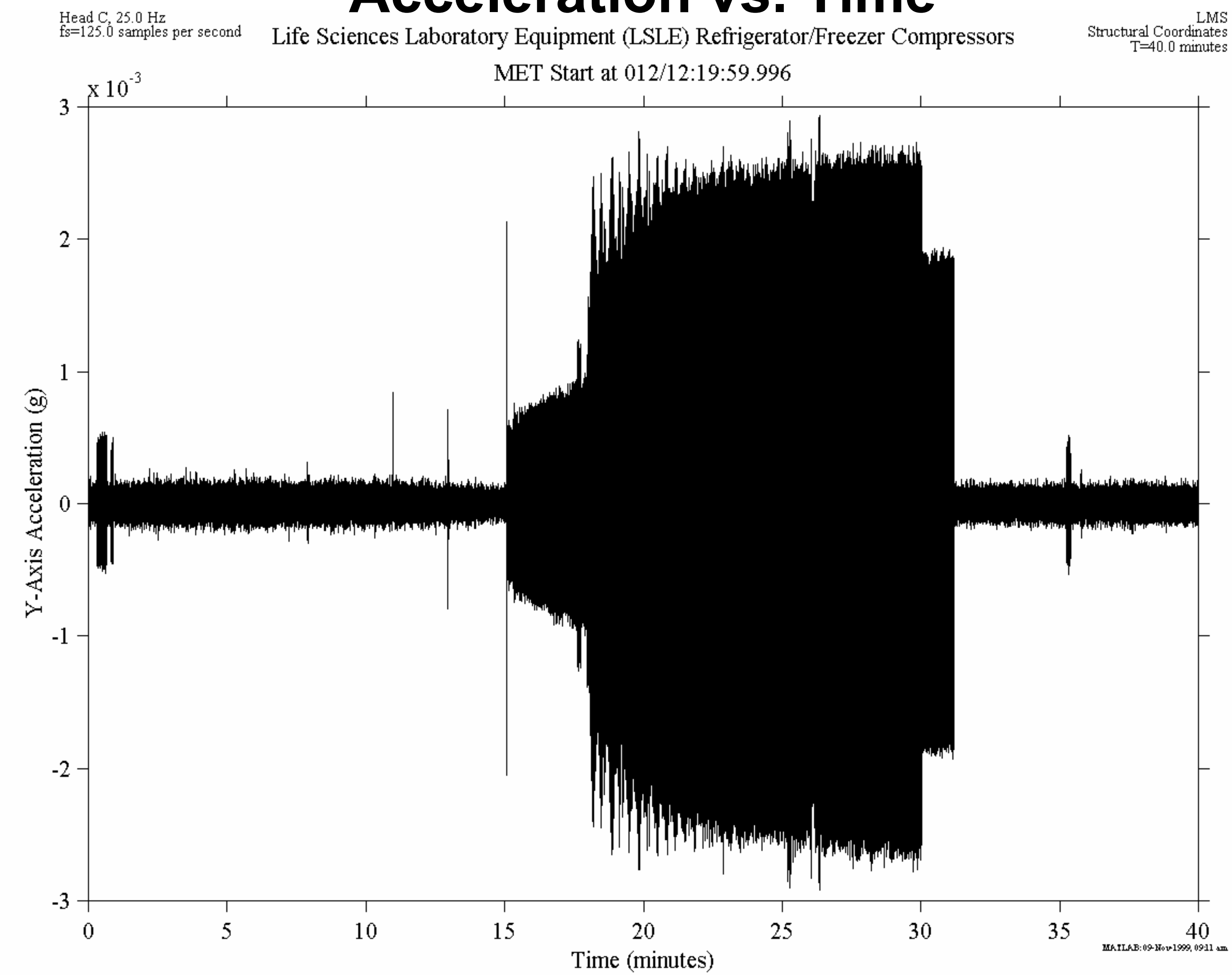
- most precise accounting of the measured data with respect to time
- fundamental approach to quantifying acceleration environment
- "purest" form of the data collected

Disadvantages:

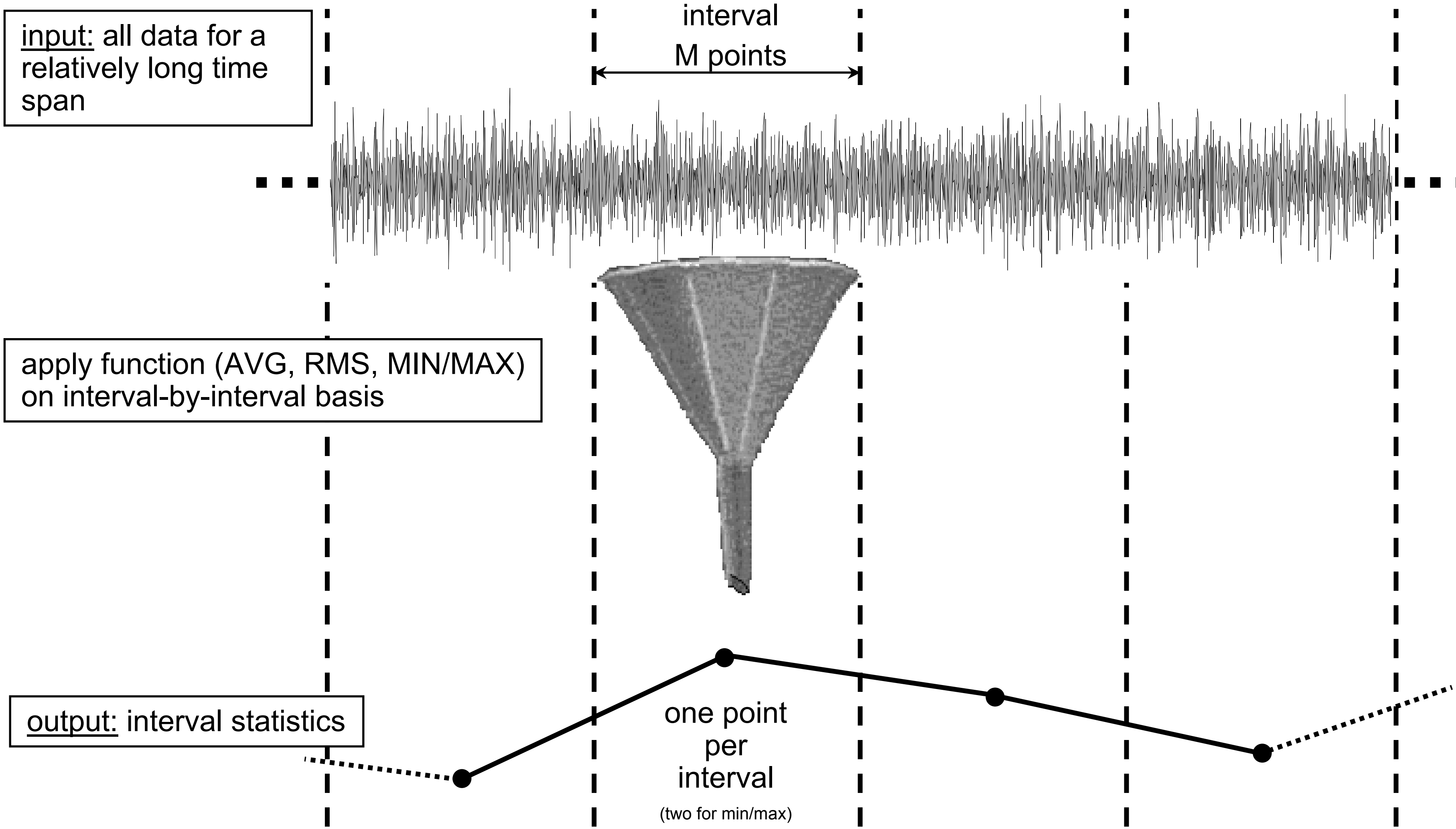
- display device (video, printer) constrains resolution for long time spans or high sample rates
- usually not good for qualifying acceleration environment ... that's the strength of frequency domain analysis

Time Domain Analysis

Acceleration vs. Time



Time Domain Analysis
Interval Processing



Time Domain Analysis
Interval AVG, RMS, MIN/MAX vs. Time

Mathematical Description:

- **AVG**: average (mean) value for each interval

$$x_{AVG}(m) = \frac{1}{M}\sum_{i=1}^{M} x((m-1)M+i); \quad m = 1, 2, ..., \left\lfloor \frac{N}{M} \right\rfloor$$

- **RMS**: root-mean-square value for each interval

$$x_{RMS}(m) = \sqrt{\frac{1}{M}\sum_{i=1}^{M} x((m-1)M+i)^2}; \quad m = 1, 2, ..., \left\lfloor \frac{N}{M} \right\rfloor$$

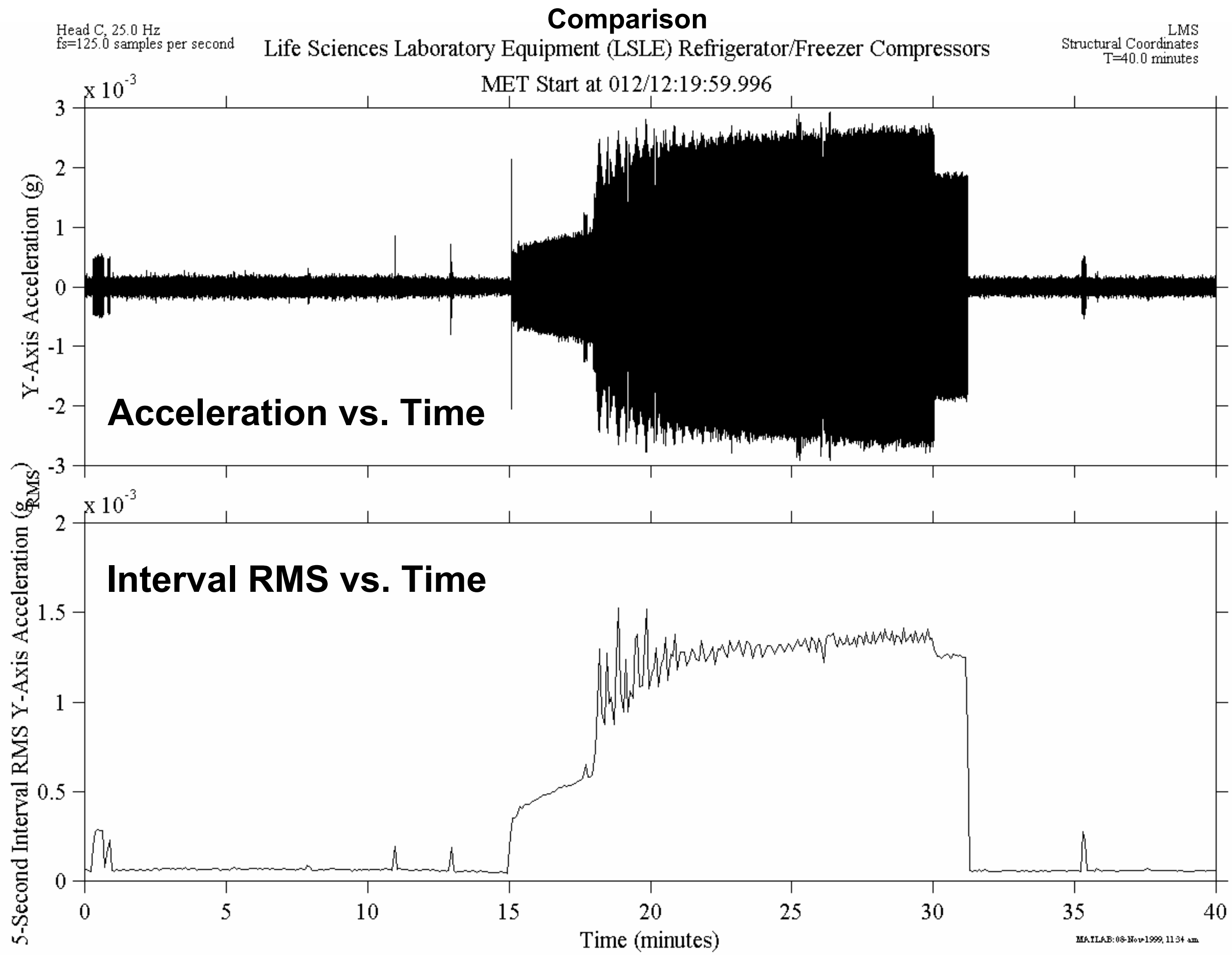
- N is number of data points that span the entire interval of interest
- M is the number of data points that span a processing interval
- m is the interval index

and $\lfloor \ \rfloor$ is the floor function

- **MIN/MAX**: both minimum and maximum values are plotted for each interval – a good display approximation for time histories on output devices with insufficient resolution to display all data in time frame of interest

Advantages:	Disadvantages:
• descriptive statistics……………………………	not-fully-descriptive statistics
• decimation (compression)......…………………	lossy

Time Domain Analysis



Frequency Domain Analysis

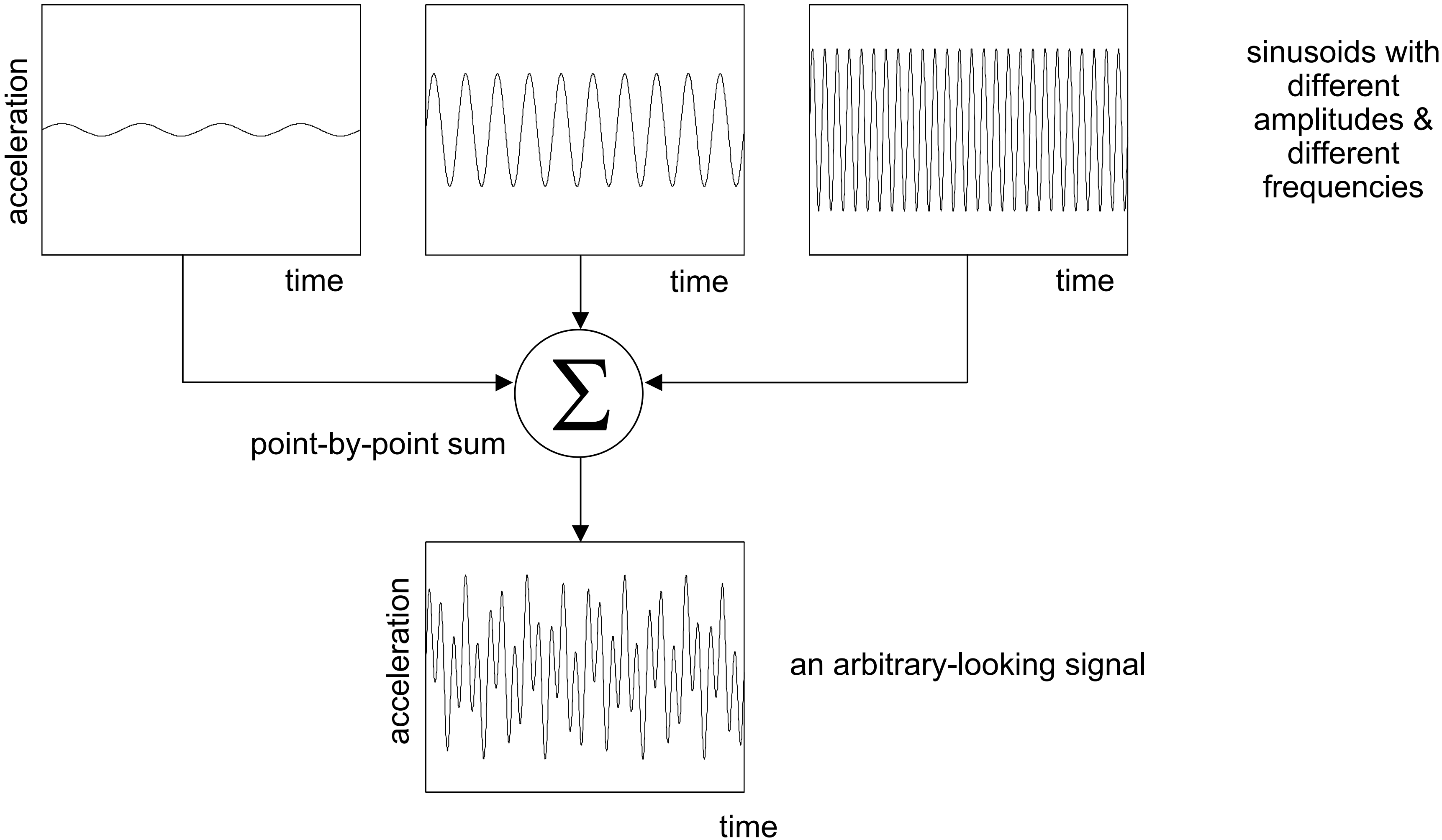
<u>Objectives:</u>

- identify and characterize oscillatory acceleration disturbances
- selectively quantify the contribution of various disturbance sources to the overall measured microgravity environment

<u>Approaches:</u>

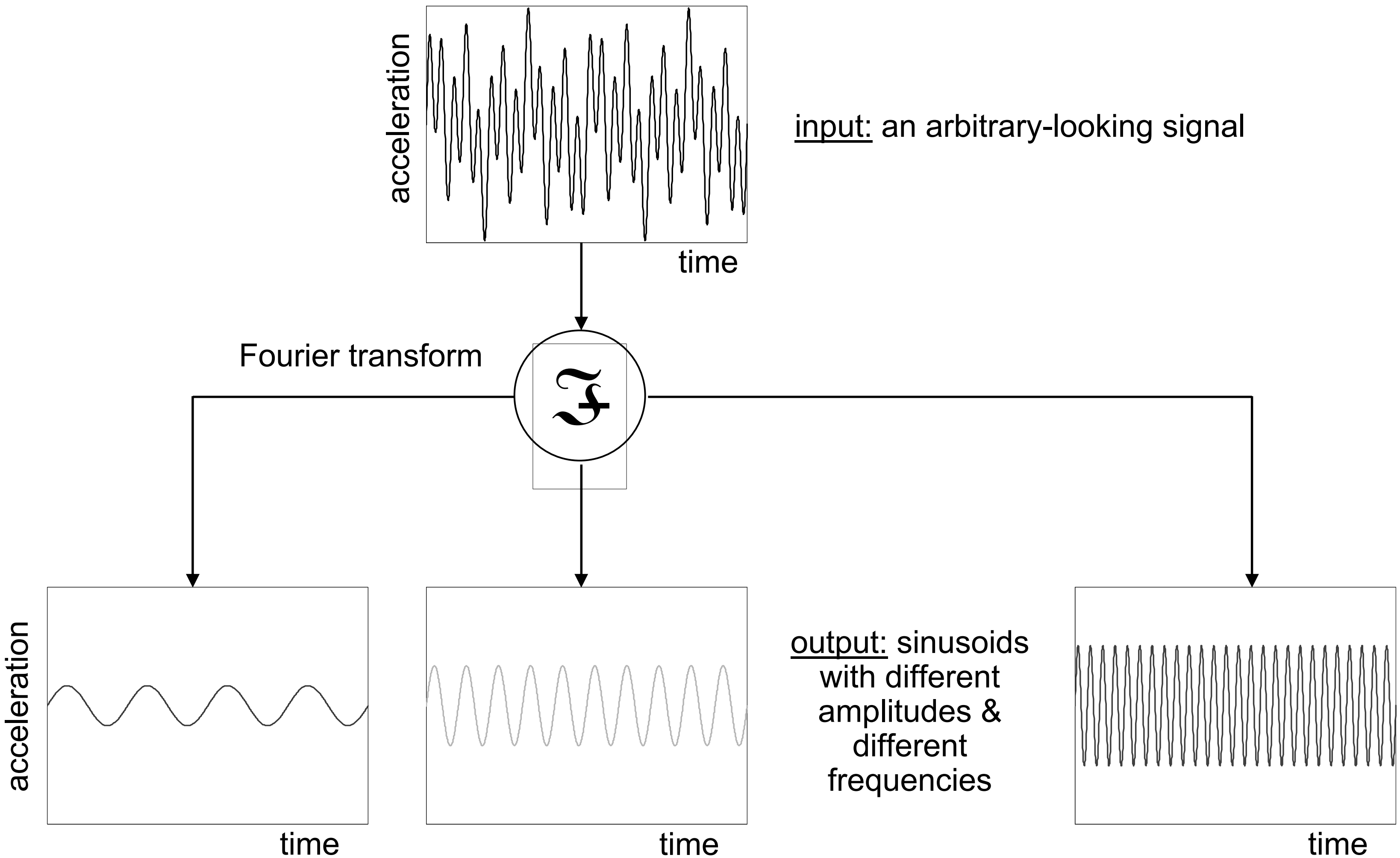
- acceleration power spectral density (PSD) → Parseval's Theorem
- cumulative RMS acceleration vs. frequency
- RMS acceleration vs. one third octave frequency bands
- acceleration spectrogram (PSD vs. *time*)
- principal component spectral analysis (PCSA) vs. frequency

Frequency Domain Analysis
Build Arbitrary-Looking Signal

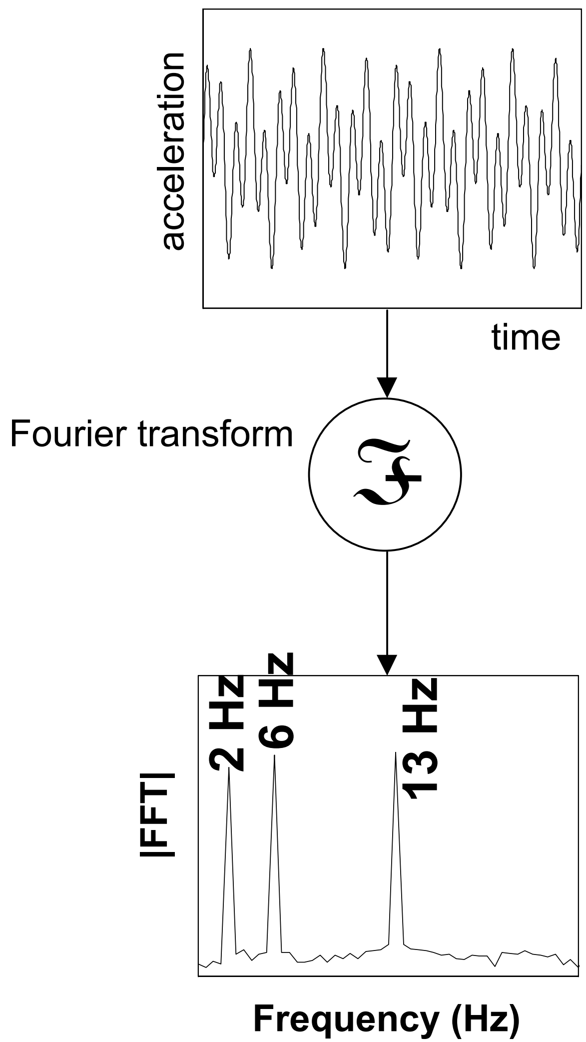


Frequency Domain Analysis

Fourier Transform: Graphical Interpretation



Frequency Domain Analysis
Fourier Transform: Graphical Description



input: an arbitrary-looking signal

output: sinusoids with different amplitudes & different frequencies

Frequency Domain Analysis

Fourier Transform: Mathematical Description

- **What is it?** It's a mathematical transform which resolves a time series into the sum of an average component and a series of sinusoids with different amplitudes and frequencies.
- **Why do we use it?** It serves as a basis from which we derive the power spectral density.
- Mathematically, for continuous time series, the Fourier transform is expressed as follows:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt\,; \;\; j = \sqrt{-1}$$

- For finite-duration, discrete-time signals, we have the discrete Fourier transform (DFT):

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \Delta t \qquad k = 0, 1, 2, \ldots, (N-1)$$

$$\Delta f = \frac{f_s}{N} = \frac{1}{T} \quad k\Delta f \qquad n\Delta t \quad \Delta t = \frac{1}{f_s}$$

- N is the number of samples in the time series
- T is the span in seconds of the time series
- f_s is the sample rate in samples/second (Hz)
- Δf is the frequency resolution or spacing between consecutive data points (Hz)

- For a power of two number of points, N, a high-speed algorithm that exploits symmetry is used to compute the DFT. This algorithm is called the fast Fourier transform (FFT). **The algorithm is different, but results of DFT and FFT are mathematically equivalent.**

Frequency Domain Analysis

Power Spectral Density (PSD): Mathematical Description

- **What is it?** It's a function which quantifies the distribution of power in a signal with respect to frequency.
- **Why do we use it?** It is used to identify and quantify vibratory (oscillatory) components of the acceleration environment.
- Mathematically, we calculate the PSD as follows:

$$P(k) = \begin{cases} \dfrac{2|X(k)|^2}{NUf_s} & [g^2/\mathrm{Hz}] \text{ for } k = 1, 2, \ldots, (N/2) - 1 \\ \dfrac{|X(k)|^2}{NUf_s} & [g^2/\mathrm{Hz}] \text{ for } \underbrace{k = 0}_{\text{DC}} \text{ and } \underbrace{k = (N/2)}_{\text{Nyquist}} \end{cases}$$

$\boxed{k\Delta f}$

$$\boxed{U = \frac{1}{N}\sum_{n=0}^{N-1} w(n)^2}$$

- X(k) is the "Δt-less" FFT of x(n)
- N is the number of samples in the time series (power of two)
- f_s is the sample rate (samples/second)
- U is window compensation factor
- w(n) is window (weighting) function

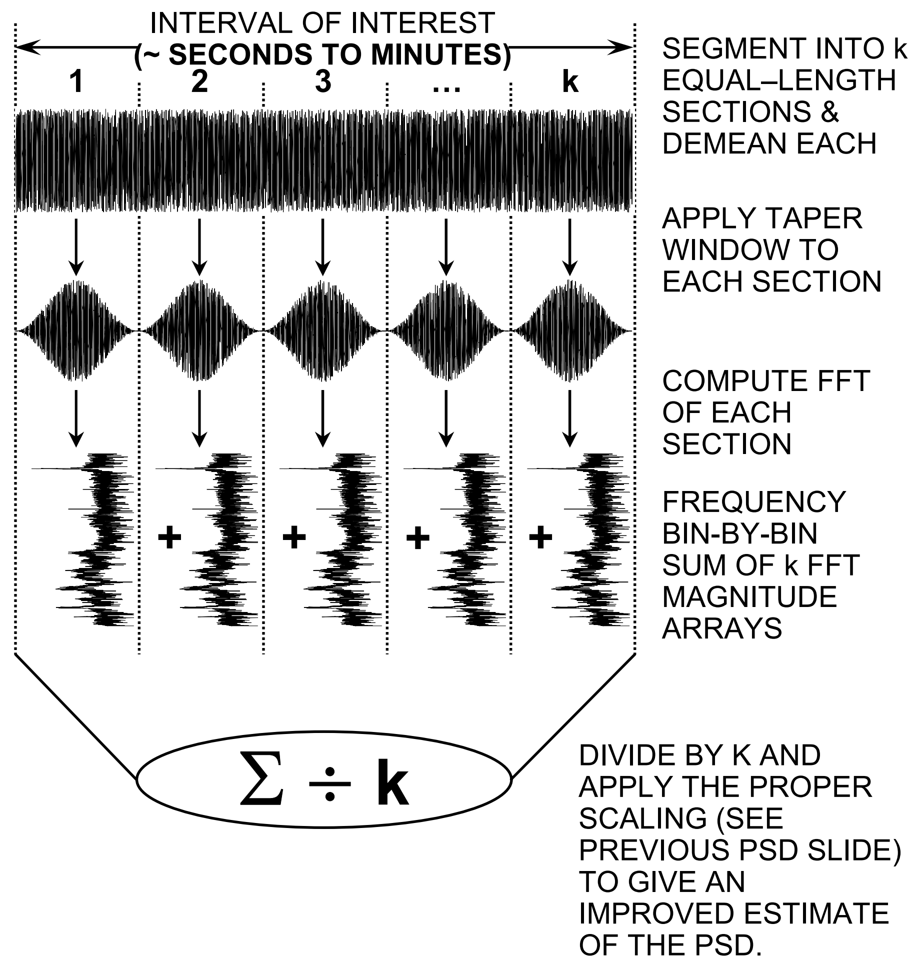
- DC is an electrical acronym for **d**irect **c**urrent that has been generalized to mean average value
- Nyquist frequency (f_N) is the highest resolvable frequency & is half the sampling rate ($f_N=f_s/2$)
- Symmetry in the FFT for real-valued time series, x(n), results in one-sided PSDs; only the DC and Nyquist components are unique – that's why no factor of 2 for those in the equation
- <u>Caution:</u> some software package PSD routines scale by some combination of f_s, 2, or N

Frequency Domain Analysis
Spectral Averaging*

- **Why? To reduce spectral variance.**
  The averaging in this process causes the variance of the PSD estimate to be reduced by a factor of k.
- **How? Welch's (periodogram) method.**
- **Tradeoff: Degraded frequency resolution.**
  As the number of averages (or sections, k) increases, the spectral variance decreases, but this comes at the expense of diminished frequency resolution. This stems from the fact that for a given time series, the more sections you have, the fewer the number of points you get in each section.

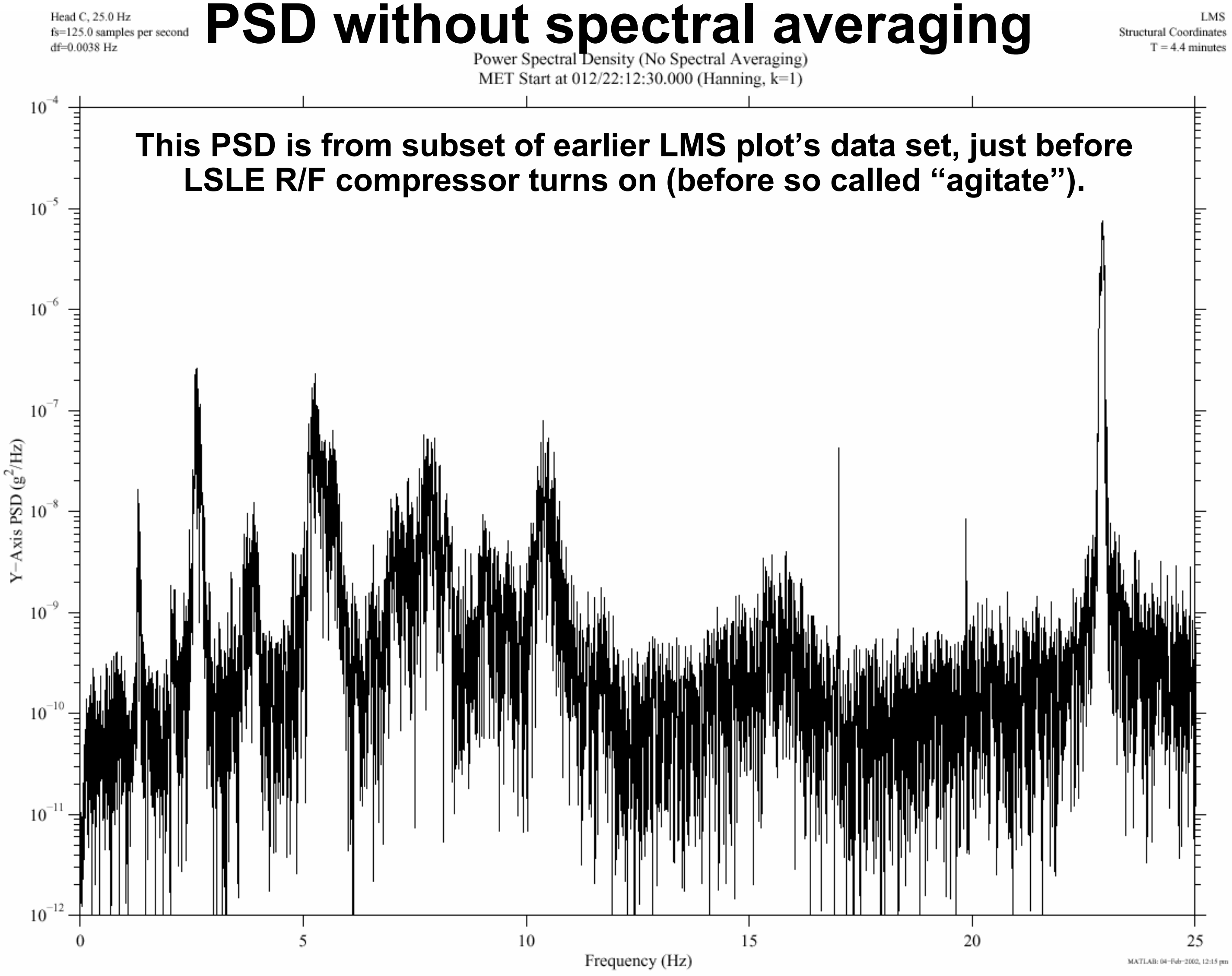
***Assumption: Data is stationary.**

Welch's (Periodogram) Method



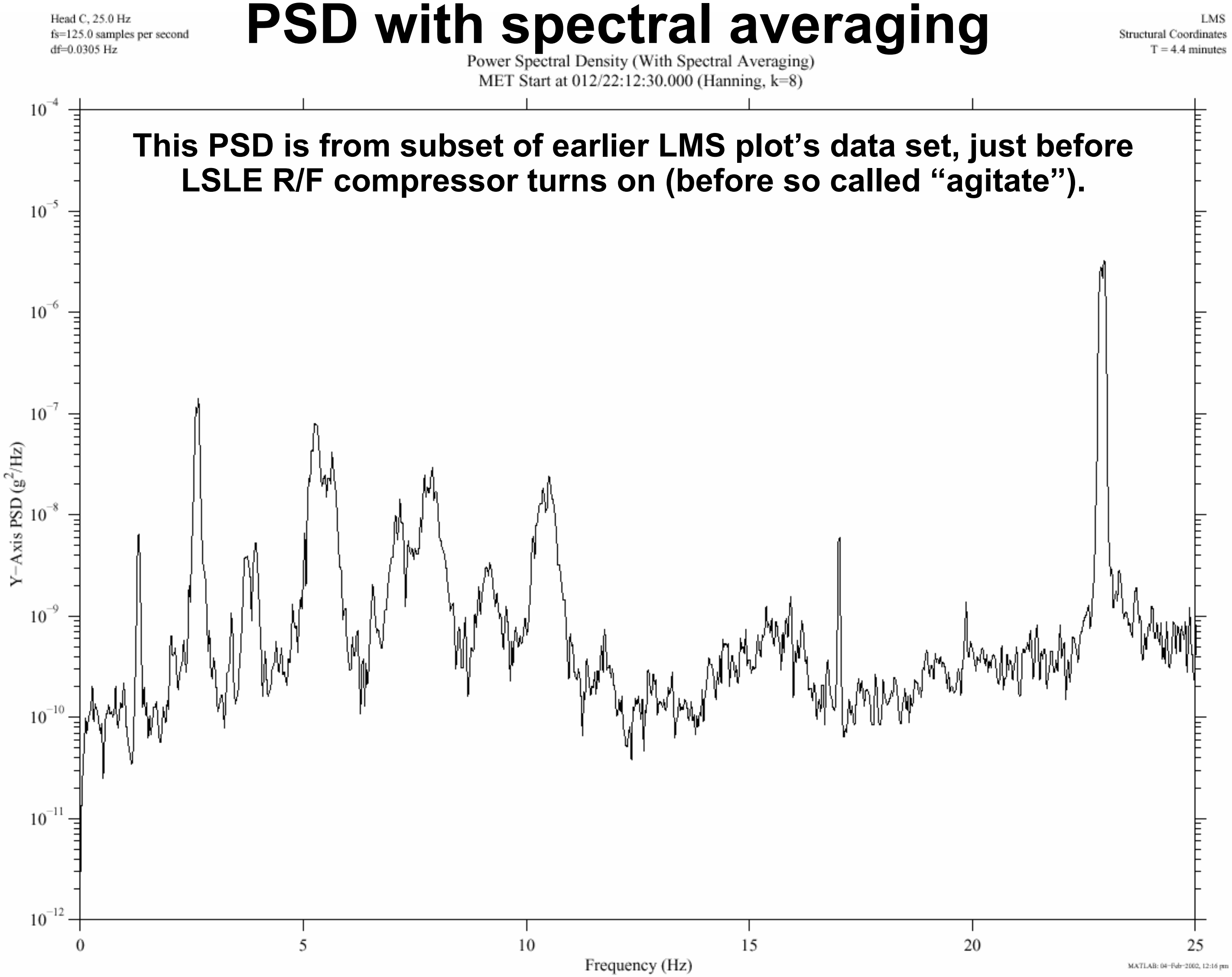
Frequency Domain Analysis

PSD without spectral averaging



Frequency Domain Analysis
PSD with spectral averaging

This PSD is from subset of earlier LMS plot's data set, just before LSLE R/F compressor turns on (before so called "agitate").



Frequency Domain Analysis

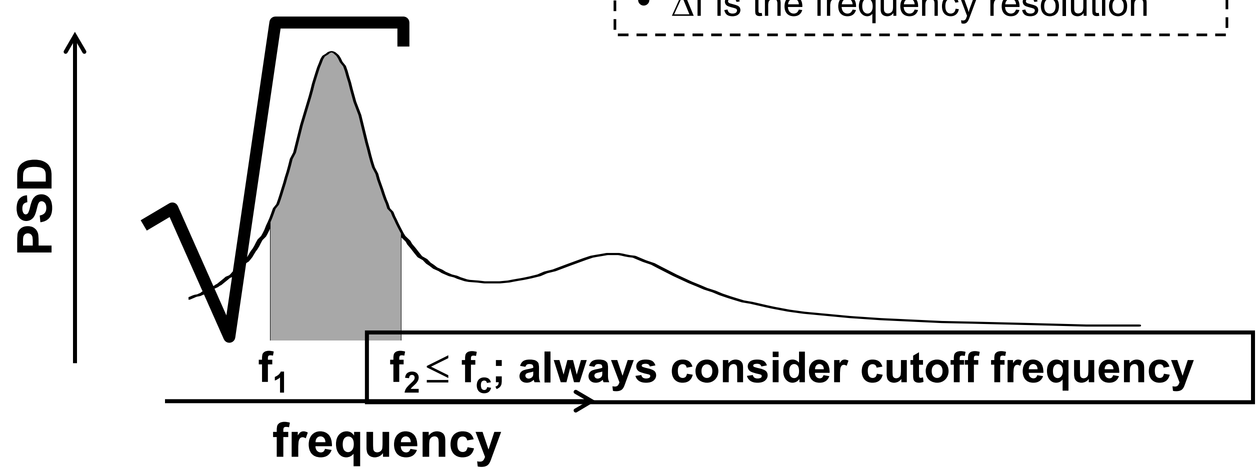
Parseval's Theorem

- **What is it?** It's a relation that states an equivalence between the RMS value of a signal computed in the time domain to that computed in the frequency domain.
- **Why do we use it?** It can be used to attribute a fraction of the total power in a signal to a user-specified band of frequencies by appropriately choosing the limits of integration (summation).
- Mathematically, this theorem can be expressed as:

$$\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}\left|x(n)\right|^2} = \sqrt{\sum_{k=0}^{N/2} P(k)\Delta f}$$

- x(n) is time series
- N is the number of samples in the time series
- P(k) is the PSD of x(n)
- Δf is the frequency resolution

$$\mathrm{RMS}\Big|_{f_1}^{f_2} =$$



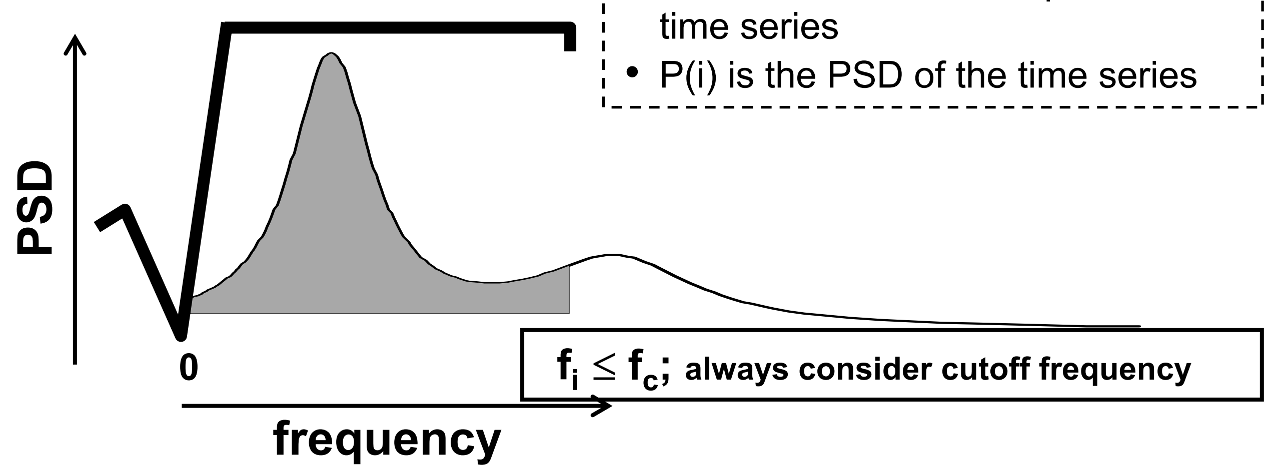
Frequency Domain Analysis
Cumulative RMS vs. Frequency

- **What is it?** It's a plot that quantifies the contributions of spectral components *at and below* a given frequency to the overall RMS acceleration level for the time frame of interest.
- **Why do we use it?** This type of plot highlights, in a quantitative manner, how various portions of the acceleration spectrum contribute to the overall RMS acceleration level.
  - steep slopes indicate relatively strong narrowband disturbances
  - shallow slopes indicate relatively quiet, broadband portions of the spectrum
- Mathematically, we have:

$$a_{RMS}(k) = \sqrt{\sum_{i=0}^{k} P(i)\Delta f} \qquad k = 0,1,2...,(N/2)$$

- Δf is the frequency resolution
- N is the number of samples in the time series
- P(i) is the PSD of the time series

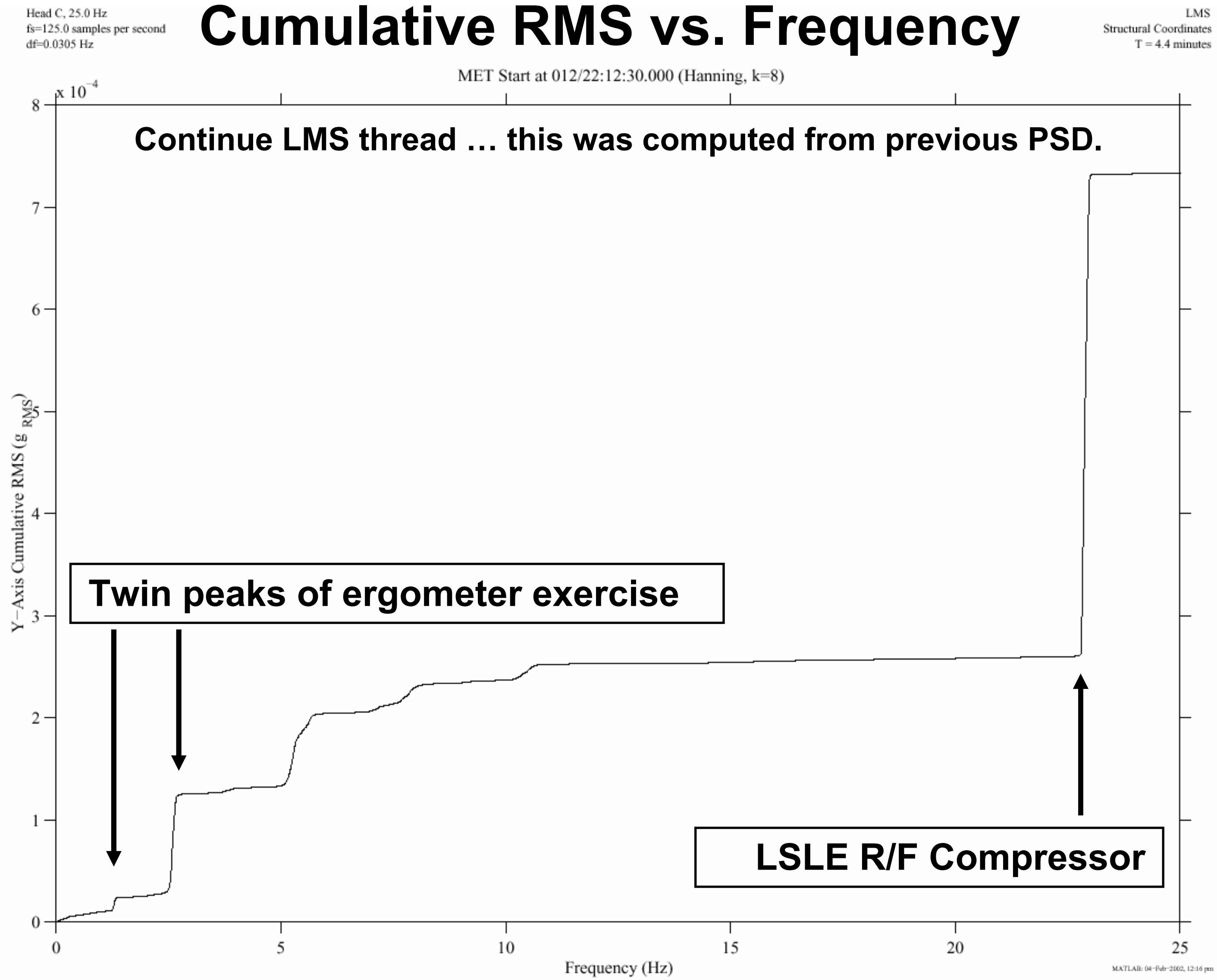
$$RMS\Big|_{0}^{f_i} =$$



lower limit is clamped to zero, while upper limit slides to right (higher frequenices)

Frequency Domain Analysis

Cumulative RMS vs. Frequency



Frequency Domain Analysis

RMS vs. One Third Octave Frequency Bands

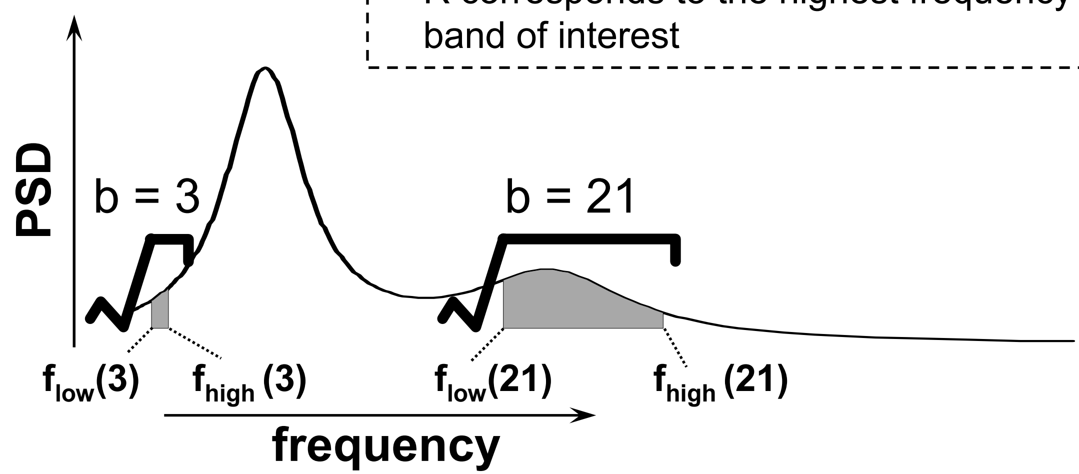
- **What is it?** It's a plot that quantifies the spectral content in proportional bandwidth frequency bands for a given time interval of interest.
- **Why do we use it?** The International Space Station vibratory limit requirements are defined in terms of the RMS acceleration level for each of a few dozen one third octave bands between 0.01 & 300 Hz with specified interval of 100 seconds.

- Mathematically, we have:

$$a_{RMS}(b) = \sqrt{\sum_{i=f_{low}(b)}^{f_{high}(b)} P(i)\Delta f} \qquad b = 1,2...,R$$

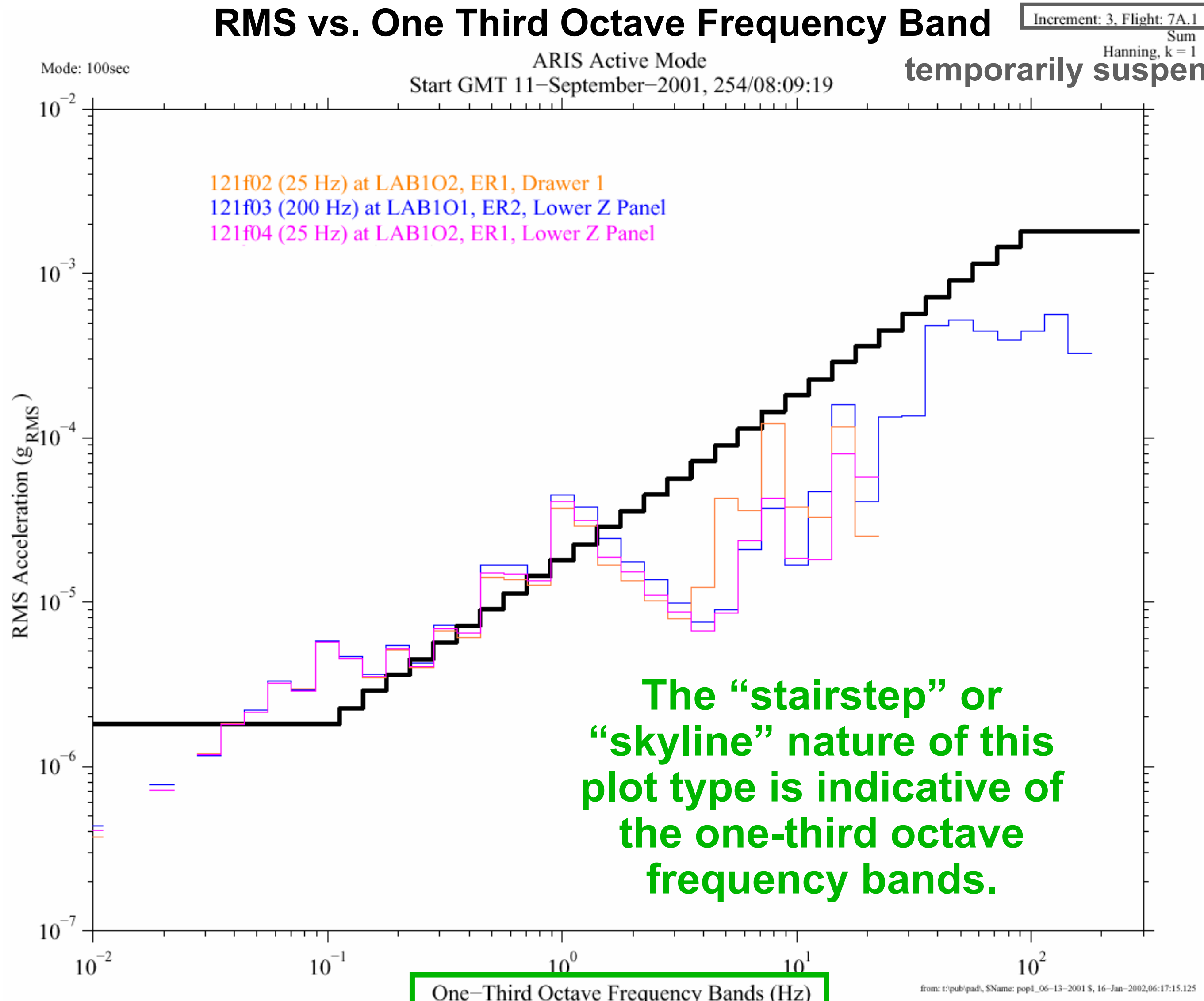
- $f_{low}(b)$ and $f_{high}(b)$ are frequency indices for the b^{th} one third octave band
- P(i) is the PSD of the time series
- Δf is the frequency resolution
- R corresponds to the highest frequency band of interest

$$RMS_b\Big|_{f_{low}(b)}^{f_{high}(b)} =$$



Frequency Domain Analysis

RMS vs. One Third Octave Frequency Band



temporarily suspend LMS thread

Log scale

The "stairstep" or "skyline" nature of this plot type is indicative of the one-third octave frequency bands.

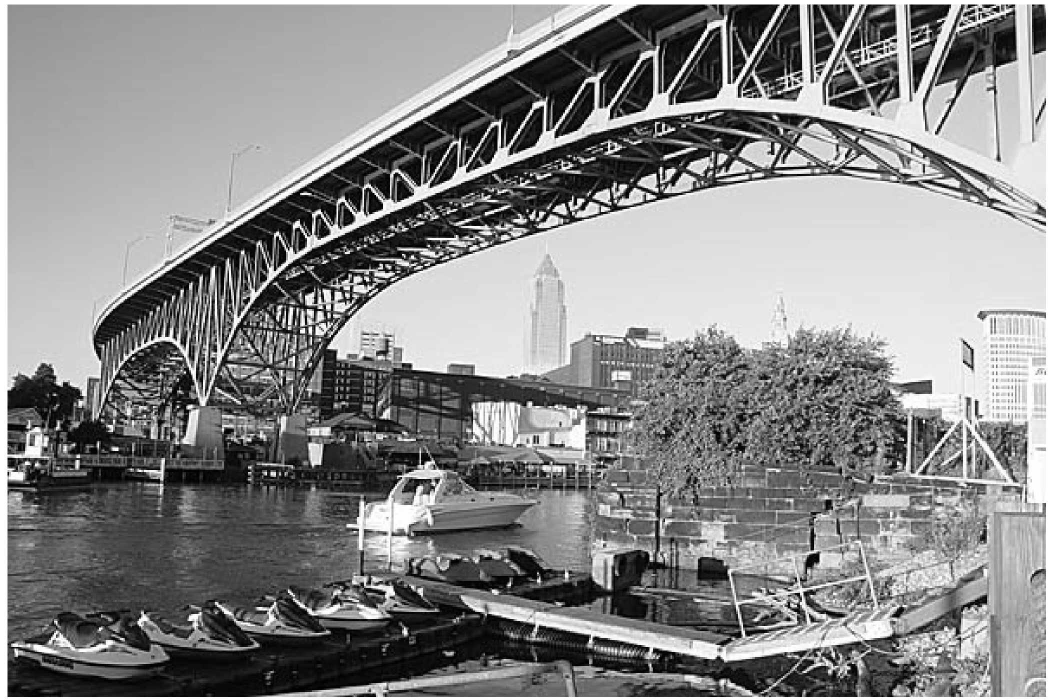
Log scale

Frequency Domain Analysis Motivation

Approximate stack height of SAMS/MAMS CDs compiled for 15-year ISS life span is about same length as depth of NASA GRC Zero-G Facility (ZGF) **~475 feet**

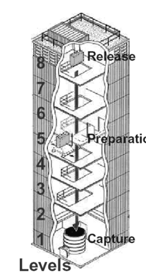
You won't see this red mark on the poster at ZGF, it was inserted in this figure for comparison.

The CD stack height would fill the ZGF shaft.



Cleveland's Main Avenue Bridge **~ 92 feet**

NASA GRC 2.2-Second Drop Tower **~ 79 feet**

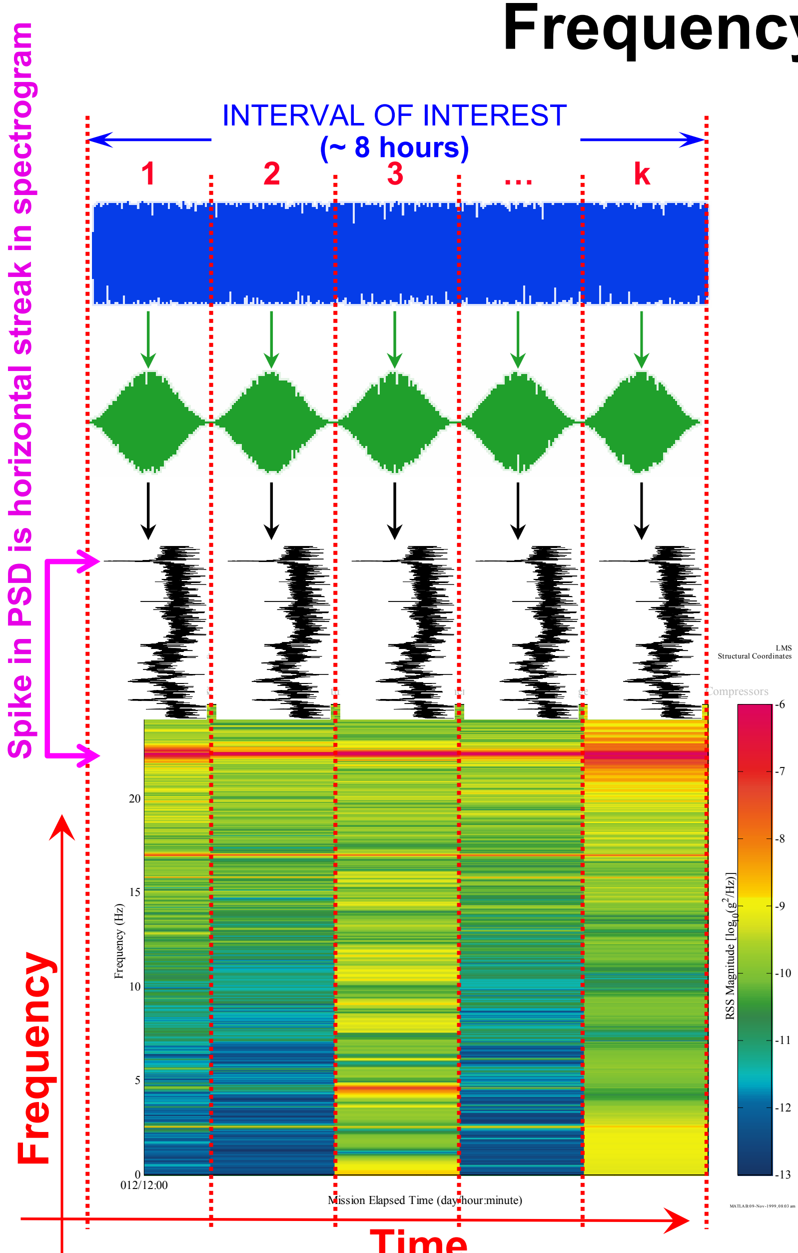


0 feet

Frequency Domain Analysis
Spectrogram

- **What is it?** A spectrogram is a three-dimensional plot that shows PSD magnitude (represented by color) versus frequency versus time.
- **Why do we use it?**
  - It is a powerful qualitative tool for characterizing long periods of data
  - Identification and characterization of boundaries and structure in the data
    - Determine start/stop time of an activity within temporal resolution, dT (if overlap, then dT is not Δt)
    - Track frequency characteristics of various activities within frequency resolution, Δf
- **Things you should NOT do with a spectrogram:**
  - Quantify disturbances in an absolute sense. The cumulative RMS or one-third octave versus frequency plots are better suited for this objective.
  - Rely entirely on it to check for the presence of a disturbance which is either known or expected to be relatively weak (instead, use PSD with proper spectral averaging or PCSA, defined later).

Frequency Domain Analysis



How to Build a Spectrogram

1. Segment demeaned data into k equal–length sections.

2. Apply taper window to each section.

3. Compute FFT of each tapered section & apply PSD scaling.

4. Calculate logarithm of PSD slices and map numeric results to color such that bottom of color scale (blue end) represents smaller values than those toward top (red end).

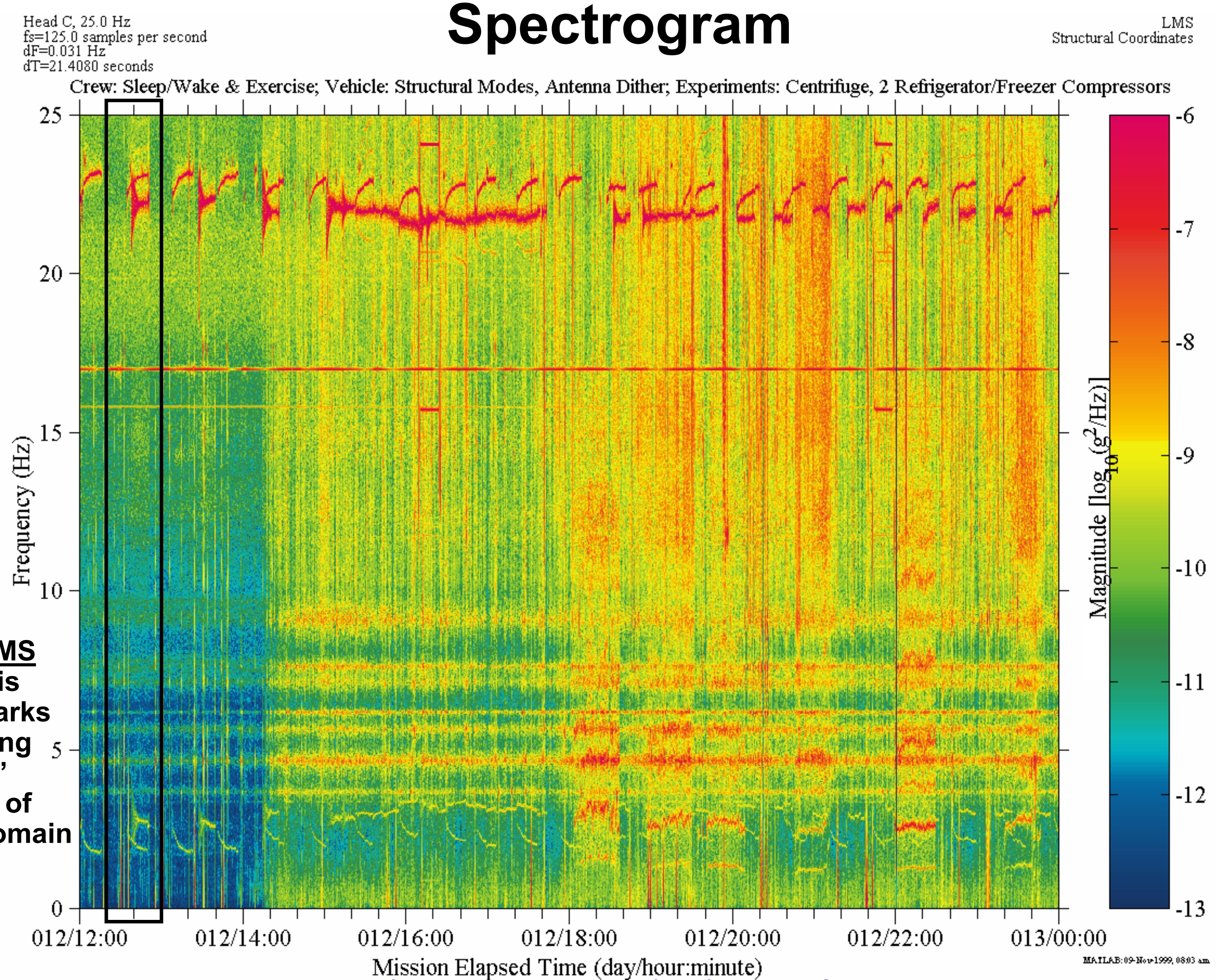
5. Display each of the k PSD slices as a vertical strip of the spectrogram (like wallpaper), such that time increases from left to right and frequency increases from bottom to top.

Note: The width of each strip is the temporal resolution & height of each distinct color patch is frequency resolution. ← This example has poor temporal resolution & good frequency resolution.

PSD Magnitude

Frequency Domain Analysis

Spectrogram



<u>continue LMS thread</u>: this rectangle marks the "washing machine" time frame of earlier time domain plot

boundaries & structure in time & frequency

Frequency Domain Analysis

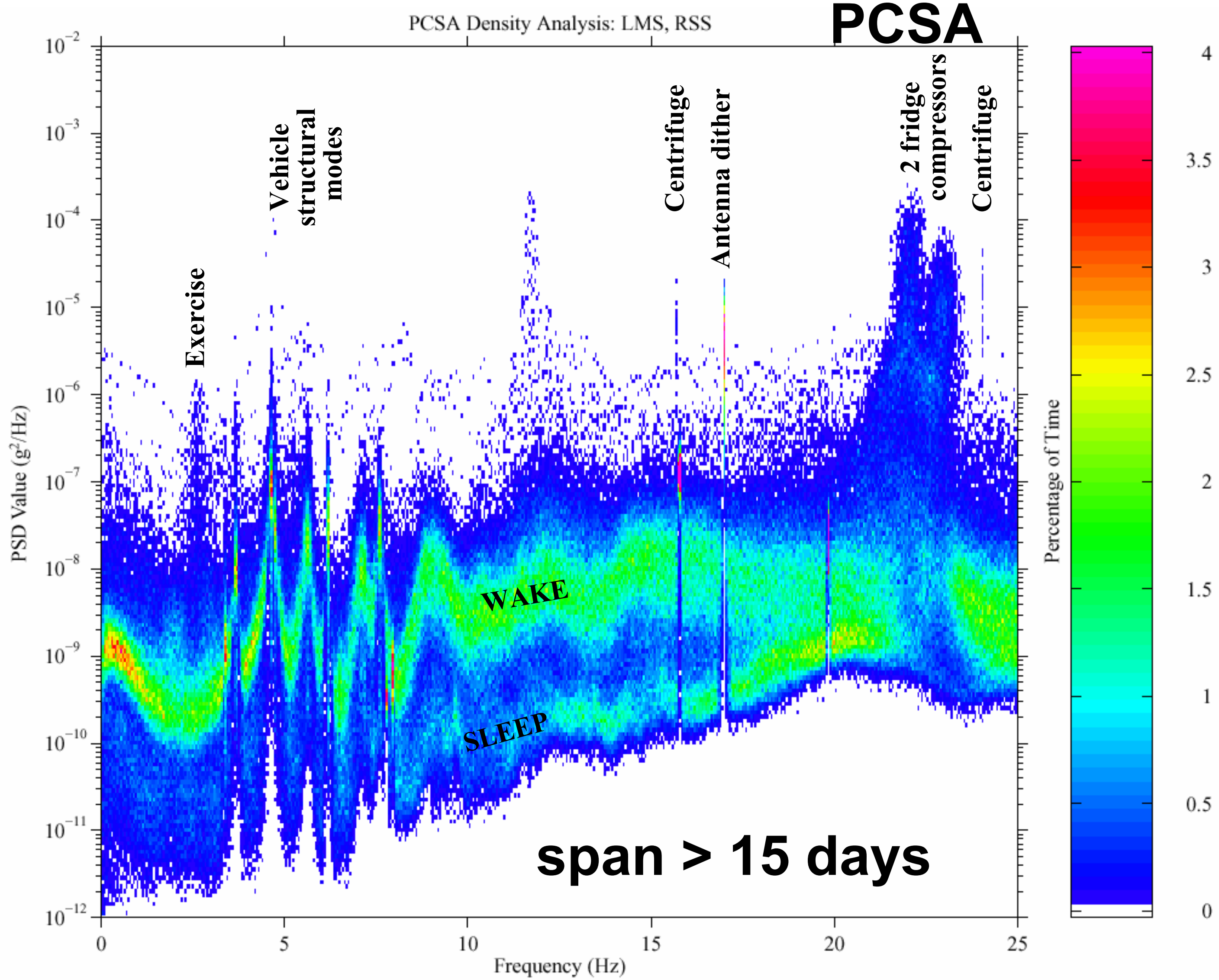
Principal Component Spectral Analysis (PCSA)

- **What is it?** A frequency domain analysis technique that compiles PSDs in the form of a two-dimensional histogram with frequency-magnitude bins.

- **Why do we use it?** To examine the spectral characteristics of a long period of data.
  - summarize magnitude and frequency variations of key spectral contributors
  - better PSD magnitude resolution relative to a spectrogram

Tradeoff: Poor temporal resolution

Frequency Domain Analysis



Time Domain Summary Table

<table>
<tr><th>DISPLAY</th><th>NOTES</th></tr>
<tr><td>Acceleration vs. Time</td><td>• most precise accounting of measured data with respect to time
• display device constrains resolution for long time spans or high sample rates</td></tr>
<tr><td>Interval Minimum/Maximum Acceleration vs. Time</td><td>• displays upper and lower bounds of peak-to-peak excursions
• good display approximation for time histories on output devices with resolution insufficient to display all data in time frame of interest (see notes below though)</td></tr>
<tr><td>Interval Average Acceleration vs. Time</td><td rowspan="2">• descriptive statistics
• not fully descriptive (“lossy compression”)</td></tr>
<tr><td>Interval Root-Mean-Square (RMS)
Acceleration vs. Time</td></tr>
</table>

Frequency Domain Summary Table

DISPLAY	NOTES
Power Spectral Density (PSD) vs. Frequency	• quantifies distribution of power with respect to frequency • windowing (tapering) to suppress spectral leakage • spectral averaging to reduce spectral variance (degraded Δf)
Cumulative RMS Acceleration vs. Frequency	• quantifies RMS contribution at and below a given frequency • quantitatively highlights key spectral contributors
RMS Acceleration vs. One Third Octave Frequency Bands	• quantify RMS contribution over proportional frequency bands • compare measured data to ISS vibratory requirements
Spectrogram (PSD vs. Frequency vs. Time)	• displays power spectral density variations with time • good *qualitative* tool for characterizing long periods • identify structure and boundaries in time and frequency
Principal Component Spectral Analysis (PCSA)	• summarize magnitude and frequency excursions for key spectral contributors over a long period of time • results typically have finer frequency resolution and high PSD magnitude resolution relative to a spectrogram at the expense of terrible temporal resolution